Identifying S-Shaped Consumption Utility and Solving the Equity Premium Puzzle

Abstract

We identify the S-Shaped consumption utility by reconciling consumption decisions with asset returns. Different from the concave-shaped utility, the S-shaped consumption utility predicts a possible negative correlation between low quantiles of consumption growth and asset returns, for which we find evidence from micro-level consumption data. The "partial" negative correlation accounts for the low correlation between consumption growth and asset returns, which is at the heart of many pricing puzzles. Using an estimable asset pricing model built upon micro consumption, we show that the S shape and the partial negative correlation quantitatively solve the equity premium puzzle.

Key words: S-Shaped Consumption Utility, Asset Pricing, Equity Premium Puzzle, Micro Consumption.

JEL classification: G12, D91, D12, E21, D15, G41

1 Introduction

This paper shows that asset pricing based on micro consumption helps identify S-shaped consumption utility and that the S-shaped consumption utility solves the equity premium puzzle. We empirically find that asset returns correlate negatively with many individuals' low-quantile consumption growth. Using this finding and its implications for asset pricing, we demonstrate that the consumption utility functions cannot be globally concave-shaped. We show that many asset pricing puzzles root in the application of concave-shaped utility and that an S-shaped consumption utility offers a plausible solution. The estimation results of our structural model based on micro consumption data confirm that the convex part in the S-shaped utility, together with the above mentioned negative correlation, can account for the high equity premium.

When confronted with financial data, consumption-based models do not perform well. Mehra and Prescott (1985) find that consumption-based asset pricing models (C-CAPM) fail to explain the high equity premium. Weil (1989) argues that the C-CAPMs are hard to rationalize the low risk-free rate. Shiller (1981), LeRoy and Porter (1981), and Grossman and Shiller (1981) show that stock prices are too volatile to be explained by economic fundamentals, such as consumption or dividends. Economists refer to these anomalies as the equity premium puzzle, risk-free rate puzzle, and equity volatility puzzle, respectively. These puzzles and many other related asset pricing puzzles discussed later in Section 2 indicate a possible giant gap in our understanding of consumption behaviors. More specifically, the consumption utility functions widely used in economics may be flawed.

Economists have explored many types of utility functions attempting to solve the asset pricing puzzles, including utility with inner or external habits (e.g., Constantinides 1990, Abel 1990, and Campbell and Cochrane 1999), recursive utility (e.g., Epstein and Zin 1989, Bansal and Yaron 2004), ambiguity aversion (e.g., Ju and Miao 2012), reference-dependent preferences (e.g., Benartzi and Thaler 1995, Barberis et al. 2001, Yogo 2008, Routledge and Zin 2010, and Barberis et al. 2015), to name a few. However, economists do not reach a consensus on the consumption utility functional form, and asset pricing puzzles are still largely unsolved after four decades of research.

An alternative that the literature has not explored is the S-shaped consumption utility. Kahneman and Tversky's (1979) experimental evidence suggests that people's utility function is S-shaped. Ju and Li (2021) present empirical findings from micro consumption data that support the S-shaped consumption utility. However, the belief that the consumption utility function is concave-shaped is deeply ingrained in the extant studies. The literature of the past four decades sees virtually no application of the S shape in modeling consumption behaviors. To justify the S-shaped consumption utility, one needs (1) provide empirical evidence and convincing reasoning that reject the concave-shaped consumption utility and (2) solve the consumption-based asset pricing puzzles using the S-shaped consumption utility. We accomplish our goals using a qualitative model and an estimable structural model.

The qualitative asset pricing model based on an S-shaped consumption utility suggests that the correlation between an individual's consumption growth and asset returns can be negative. The negative correlation is not a prediction of concave-shaped consumption utility. When an individual with a concave-shaped consumption utility function plans to lower her marginal utility, she must raise her consumption. In contrast, an individual with an S-shaped consumption utility has two types of choices to reduce her marginal utility; she can either increase or decrease her consumption. Given a positive risk premium, the correlation between risky returns and the marginal utility of consumption is negative. Therefore, the correlation between the risky returns and consumption growth of an individual with a concave-shaped utility is positive; however, the correlation can be positive or negative for individuals with an S-shaped consumption utility. The negative correlation arises because of the convex section of the S-shaped consumption utility. An individual decreases her consumption when the asset return rises because the loss of utility can be compensated or over-compensated by consuming more in the future.

Finding the negative correlation between consumption growth and risky returns will reject the globally concave-shaped consumption utility. The negative correlation implies that the risky asset serves as "insurance" against consumption risks. Individuals with a concave-shaped consumption utility demand a negative risk premium of the "insurance". However, of the same asset, the remaining risk-averse individuals command a positive risk premium because their consumption growth correlates positively with the risky return. We, therefore, arrive at a contradiction because the law of one price does not hold.

We provide empirical evidence from micro consumption data rejecting the concave-shaped utility. The challenge in identifying the above negative correlation is that there are also individuals for whom the correlation is positive. The correlation for any single individual can be positive or negative, depending on her specific situation. Our identification strategy takes advantage of a particular implication of the S-shaped consumption utility. The qualitative model predicts that low quantiles of consumption growth are more likely to correlate negatively with asset returns than the top quantiles. We confirm this prediction by running a simple quantile regression. For some subgroups, we find a significantly negative correlation between low quantiles of consumption growth and asset returns (including equity returns and risk-free returns).

Our qualitative asset pricing model and the above initial empirical evidence help explain asset pricing puzzles. First, our empirical results contribute to the understanding of the fundamental problem in asset pricing. Presumably, most economists would agree that many consumption-based asset pricing puzzles link closely to the low covariance between consumption growth and asset returns (see Cochrane and Hansen 1992, Mehra and Prescott 2003, and Campbell 2003). The established negative correlation accounts for the low covariance if it cancels out the positive correlation to a large extent (shown later in the structural model estimation). Second, the qualitative model explains how the equity premium puzzle arises. This model predicts that the individuals whose consumption growth correlates negatively with equity returns are risk-seeking toward consumption risks. They demand a positive equity premium because stocks serve against their will by hedging against the consumption risks. If researchers mistakenly model the risk seekers' behaviors using the concave-shaped consumption utility, the equity premium they demand becomes negative. Therefore, the generated aggregated equity premium can be much smaller than the observed one, even if the degree of relative risk aversion is large.

Our structural model estimation shows that the S-shaped consumption utility quantitatively solves both the equity premium puzzle and the risk-free rate puzzle. The model builds upon micro consumption and reference-dependent preference. We restrict the gain functions to be concave and the loss functions to be convex. The reference is a latent random variable. We estimate the utility function and the conditional distribution of reference points by minimizing the pricing errors of the risk-free rate and excess return rates of Fama and French's 100 portfolios sorted by size and book-to-market ratio. The estimation shows that the S-shaped consumption utility performs very well in explaining asset prices. First, our model accounts for a large proportion (91.80%) of the market equity premium. The estimated degree of relative risk aversion is a function of the consumption-reference ratio,[1] lying within a reasonable range. It is smaller than two with a probability of 98.36%. In sharp contrast to existing models in the literature, the average individual in our model has a probability close to 50% being (locally) risk-seeking. The success in explaining the high equity premium indicates that the proportion of stockholders who respond negatively to the changes in equity returns is significant. Second, the model fits the risk-free rate almost perfectly. Therefore, the S-shaped consumption utility also helps solve the risk-free rate puzzle (see Weil, 1989). The model predicts a low risk-free interest rate because many individuals save. Third, the estimated volatility of the pricing kernel is high, and Hansen and Jagannathan's (1991) inequalities hold for all of the test assets. We also estimate an alternative model that restricts the gain-loss utility function to be globally concave. Even though its performance is equally good in fitting the risk-free rate and the market equity premium, the estimated degree of relative risk aversion given a high consumption-reference ratio is absurdly large.

[1] The consumption-reference ratio refers to the consumption divided by the reference level.

2 Key problems in C-CAPMs with concave utility

We will first review the literature and summarize the fundamental problems of traditional asset pricing models based on concave-shaped consumption utility. The main conclusions are two-fold. Firstly, many consumption-based asset pricing puzzles link closely to the low covariance between per capita consumption growth and stock returns. Secondly, many small estimates of the intertemporal elasticity of substitution in the literature indicate a low correlation between consumption growth and interest rates, which challenges many extant C-CAPMs and deserves further study.

We consider a simple C-CAPM in which a representative agent maximizes her life-time expected utility:

$$E\left[\sum_{t=0}^{\infty} \beta^t U(c_t)|I_0\right], \qquad 0<\beta<1, \tag{1}$$

where c_t is her consumption; I_t is her information set at time t; and β is the subjective time discount factor. The utility function $U(\cdot)$ is concave-shaped. For expositional simplicity, we consider a constant relative risk aversion utility function $U(c_t)=\frac{c_t^{1-\gamma}-1}{1-\gamma}$, where γ measures the degree of relative risk aversion. The asset pricing equations are

$$E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^e_{t+1}|I_t\right]=1, \tag{2}$$

and

$$E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} R^f_t|I_t\right]=1, \tag{3}$$

where R^e_{t+1} and R^f_t denote the equity return rate and risk-free return rate, respectively. The term $\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ is called the stochastic discount factor (SDF) or the pricing kernel.

To illustrate the pricing puzzles, we closely follow Hansen and Singleton (1983), Mehra and Prescott (2003), and Campbell (2003) to derive, from (2) and (3), the relationships between the logarithms of consumption growth and asset returns. Let $g_{t+1}=log(c_{t+1}/c_t)$, $r^e_{t+1}=log(R^e_{t+1})$, $r^f_t=log(R^f_t)$, $Y_t=(g_t,r^e_t,r^f_t)'$, and ψ_t denote the information about

$\{Y_{t-s}\}_{s=0}^{\infty}$. We assume that Y_t is a stationary Gaussian process. Thus, we have

$$2E(r_{t+1}^{e}-r_{t}^{f}|\psi_t)+var(r_{t+1}^{e}|\psi_t)=2\gamma cov(g_{t+1},r_{t+1}^{e}|\psi_t), \tag{4}$$

$$r_{t}^{f}=\gamma E(g_{t+1}|\psi_t)-\frac{\gamma^2 var(g_{t+1}|\psi_t)}{2}-log\beta. \tag{5}$$

Equation (4) relates the log risk premiums and the (conditional) variance of stock returns to the covariance between consumption growth and stock returns. As $cov(g_{t+1},r_{t+1}^{e}|\psi_t)$ is extremely small, γ must be absurdly large to make Equation (4) hold. Thus, the *equity premium puzzle* and the *equity volatility puzzle* arise from the low covariance between consumption growth and stock returns.

Equation (5) can illustrate the *risk-free rate puzzle*. The second term at its right-hand side represents a precautionary savings effect. When per capita consumption is used, the conditional variance $var(g_{t+1}|\psi_t)$ is so small that the precautionary savings effect is negligible compared to the first term in Equation (5). Thus, the risk-free rate generated by the model increases as γ rises. To explain the equity premium, the model needs a large γ. Consequently, the model produces a much larger risk-free rate than observed in the market. Therefore, the *risk-free rate puzzle* is associated with the low variance of per capita consumption growth.

The low covariance between consumption growth and stock returns is also crucial to understanding the *stock holding puzzle* and *predictability puzzle*. Given the high equity premium, Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995) think it is puzzling that very few people participate in the stock market. Fama and French (1988) and Campbell and Shiller (1988) find a perplexing fact that price-dividend ratios predict long-horizon stock prices. These anomalies are dubbed the *stock market participation puzzle (or stock-holding puzzle)*, and *long-run stock return predictability puzzle (or predictability puzzle)*, respectively. To solve the *stock holding puzzle*, we must account for the low covariance between the stock returns and nonstockholders' consumption growth. If the covariance were (positively) large, then it would not be puzzling that few people hold stocks because participating in the stock market would have amplified their consumption risks. Regarding the *predictability puzzle*, Barberis and Thaler (2003) make a convincing point: "a resolution of the *(equity) volatility puzzle* is simultaneously a resolution of the *predictability puzzle*." [2]

Moreover, the correlation between consumption growth and interest rates matters for consumption-based asset pricing. The recent developments of asset pricing models suggest that a high value of the intertemporal elasticity of substitution (IES) helps explain the pricing anomalies. For example, Bansal and Yaron (2004) and Ju and Miao (2012) set IES to 1.5 in their calibration. However, most empirical estimates of IESs are close to zero (see the meta-analysis by Havránek (2015)). Therefore, asset pricing puzzles may also be associated with the small IESs or, equivalently, the low correlation between consumption growth and interest rates.

In summary, low covariance between consumption growth and asset returns is at the heart of many pricing puzzles. Substituting micro consumption for per capita consumption in traditional C-CAPMs helps address the small-variance problem, but the low-correlation issue becomes the focus. In the following, we show that the low correlation arises from an S-shaped consumption utility and that this utility helps solve various asset pricing puzzles.

3 Asset pricing based on S-shaped consumption utility

This section presents a simple theoretical model, showing the implications of the S-shaped consumption utility for asset pricing. Specifically, we explain how this utility helps explain consumption-based asset pricing puzzles.

Consider a three-period model with heterogeneous agents indexed by $i \in \{1, \ldots, N\}$.

[2] Barberis and Thaler's (2003) argument is based on the identity for the equity return rate R^e_{t+1}:

$$R^e_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t}, \tag{6}$$

where P_t and D_t denote the equity price and dividend, respectively. From Equation (6), it is safe to say that any model that captures the large volatility in stock returns also generates a large variance in the process P_t/D_t. When P_t/D_t reverts from a larger than average value to its mean, the stock price should decrease because a C-CAPM generally does not allow P_t/D_t to forecast dividend flows.

Each period, individuals are endowed with a certain amount of consumption goods. The endowment information in every period is known to each individual in period 1. Two financial markets are available. First, individuals can invest in a risky security market. The fund raised by this security is used to finance a risky project in period 1. At a good state with probability $p \in (0,1)$, the project offers a return rate $\bar{R}_2^e$ (greater than one) to the security holders in period 2; at a bad state with probability $1-p$, its return rate is $\underline{R}_2^e$ (smaller than one). Let the random variable R_2^e denote the risky return rate. Starting from period 2, no risky project is available, and all the uncertainty in this model resolves. Second, individuals can borrow and lend in periods 1 and 2. Defaulting is prohibited. Denote by $R_t^f (t = 1,2)$ the default-free interest rate between period t to period $t+1$. Let U_{it} denote the individual i's S-shaped utility function in period $t \in \{1,2,3\}$ (Note that the main conclusions remain if we assume concave utility functions for periods 1 and/or 3.). We assume that for all $i \in \{1,\ldots,N\}$, the marginal utility $U_{it}^{'}(c_{it})$ is positive for all c_{it} in the real line. Individual i's objective function is

$$\max_{\{c_{it}\}_{t=1}^{3}} E\left[\sum_{t=1}^{3} \beta^{t-1} U_{it}(c_{it})\right],$$

where β is the subjective time discount factor and c_{it} is her consumption.

We assume that there are no corner solutions in the equilibrium. Thus, the following necessary conditions hold:

$$1 = \beta \frac{U_{i3}^{'}(c_{i3})}{U_{i2}^{'}(c_{i2})} R_2^f, \qquad \forall i. \tag{7}$$

$$1 = E\left[\beta \frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})} R_2^e\right], \quad \forall i, \tag{8}$$

$$1 = E\left[\beta \frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})} R_1^f\right], \quad \forall i. \tag{9}$$

Equation (7) indicates that the marginal utility in period 2 is proportional to that in period 3. From Equations (8) and (9), we have

$$E\left[U_{i2}^{'}(c_{i2})(R_2^e - R_1^f)\right] = 0, \quad \forall i. \tag{10}$$

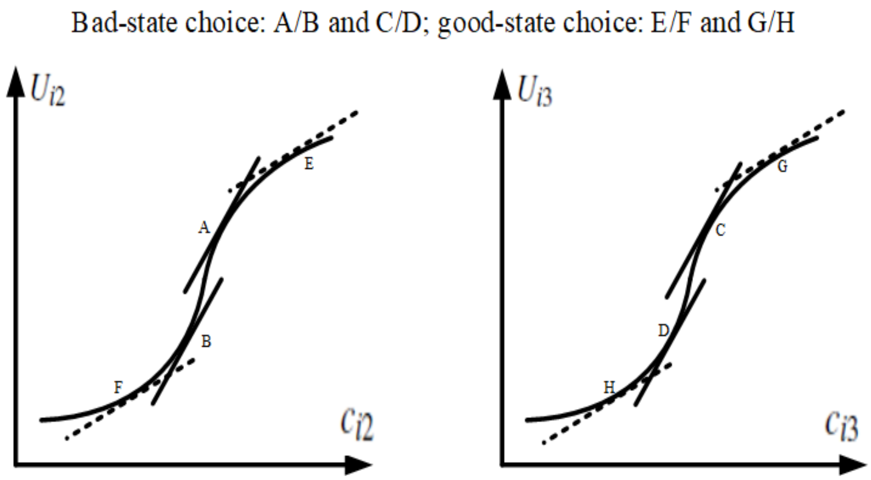

Figure 1: State-contingent consumption plans under S-shaped consumption utility

As $U_{i2}^{'}(c_{i2}) > 0$ and $\bar{R}_2^e > \underline{R}_2^e$, we obtain from Equation (10) the following inequality for the endogenously determined default-free interest rate,

$$\bar{R}_2^e > R_1^f > \underline{R}_2^e. \tag{11}$$

To discuss the properties of the marginal utility $U_{i2}^{'}(c_{i2})$, we further restrict that the risk premium be greater than zero, i.e.,

$$E[R_2^e - R_1^f] > 0. \tag{12}$$

Combining Equations (10), (11), and (12), we conclude that the marginal utility $U_{i2}^{'}(c_{i2})$ at a good state must be smaller than that at a bad state. That is,

$$U_{i2}^{'}(c_{i2})|(R_2^e = \bar{R}_2^e) \;\; < \;\; U_{i2}^{'}(c_{i2})|(R_2^e = \underline{R}_2^e), \quad \forall i.$$

In other words, for all $i \in \{1,\ldots,N\}$, the marginal utility, $U_{i2}^{'}(c_{i2})$, correlates negatively with the risky return rate, R_2^e. If individual i's utility function were concave, we would conclude that R_2^e correlates positively with consumption growth $c_{i,2}/c_{i,1}$.

However, as the utility functions are S-shaped, R_2^e may be negatively correlated with some individuals' consumption growth, whereas the correlations are positive for others. Figure 1 qualitatively illustrates how the individuals with the S-shaped utility make state-contingent consumption plans. The two sub-figures show individual i's possible consumption plans for periods 2 and 3. The tangent points between the solid lines and the S-shaped utility curves (i.e., A, B, C, and D) are her potential planned choices for the bad state ($R_2^e = \underline{R}_2^e$), and the tangent points between the dashed lines and the utility curves (i.e., E, F, G, and H) are her potential choices for the good state ($R_2^e = \bar{R}_2^e$). In period 2, her bad-state choice may be point A or B. Correspondingly, her bad-state choice in period 3 may be at point C or D. The combination (except the impossible combination (B,D)) she actually chooses is determined by her specific preference and endowments. When the realized state is $R_2^e = \bar{R}_2^e$, she implements her good-state plan. Compared with her bad-state consumption choice, she may increase her period-2 consumption, moving from point A or B to point E. Another individual with a different specification of preference and endowments may decrease her period-2 consumption, moving from point A or B to F; the reduction of consumption in period 2 is used to finance more consumption in period 3 (change from point C or D to G). Therefore, heterogeneous individuals with an S-shaped consumption utility may have opposite consumption responses to the rise of the risky return rate. Similarly, as the risky return rate falls, individuals may also have opposite consumption responses. For example, some may decrease their period-2 consumption from the good-state choice E to the bad-state choice A or B, while others may increase their period-2 consumption from the choice F to A or B. Those who increase their period-2 consumption reduce their period-3 consumption, from G to C or D.

The above different consumption behaviors associated with the S-shaped consumption utility help explain the low covariance between consumption growth and asset returns. In the literature, the widely used consumption measure is per capita consumption. Let $C_t = \sum_i c_{it}$ denote the total consumption in period t. The growth rate of the per capita consumption is a weighted average of the individual consumption growth rate, i.e.,

$$\frac{C_{t+1}}{C_t} = \frac{\sum_i c_{i,t+1}}{C_t} = \sum_i \left(\frac{c_{i,t}}{C_t}\right) \frac{c_{i,t+1}}{c_{i,t}}, \quad t = 1.$$

Denote by

$$A = \left\{ i \in \{1, \ldots, N\} : \, cov\left(\frac{c_{i2}}{c_{i1}}, R_2^e\right) < 0 \right\},$$

the set of individuals whose consumption c_{i2} (or consumption growth $\frac{c_{i2}}{c_{i1}}$) is negatively correlated with the risky return rate. Let $B = \{1, \ldots, N\}/A$ be the complement of A. We decompose the per capita consumption growth rate into two parts:

$$\frac{C_{t+1}}{C_t} = \sum_{i \in A} \left(\frac{c_{i,t}}{C_t}\right) \frac{c_{i,t+1}}{c_{i,t}} + \sum_{i \in B} \left(\frac{c_{i,t}}{C_t}\right) \frac{c_{i,t+1}}{c_{i,t}}, \quad t = 1. \tag{13}$$

In our two-state setup, the two terms in the right-hand side of equation (13) are negatively correlated. Thus, it is possible that $var(\frac{C_{t+1}}{C_t})$ is much smaller than $var(\frac{c_{i,t+1}}{c_{i,t}})$ for all $i \in \{1, \ldots, N\}$. Also, we have the following decomposition of the covariance between $\frac{C_{t+1}}{C_t}$ and R_{t+1}^e:

$$cov\left(\frac{C_{t+1}}{C_t}, R_{t+1}^e\right) = \sum_{i \in A} \left[\left(\frac{c_{i,t}}{C_t}\right) cov\left(\frac{c_{i,t+1}}{c_{i,t}}, R_{t+1}^e\right)\right] + \sum_{i \in B} \left[\left(\frac{c_{i,t}}{C_t}\right) cov\left(\frac{c_{i,t+1}}{c_{i,t}}, R_{t+1}^e\right)\right], \quad t = 1. \tag{14}$$

The first term of the right-hand side of Equation (14) is negative, and the second term is positive. If the first term largely cancels out the second one, the covariance between per capita consumption growth and the risky return rate can be close to zero. Therefore, consumption aggregation is one fundamental problem that is responsible for various consumption-based asset pricing puzzles.

Using disaggregated consumption data but applying concave utility instead of S-shaped utility also leads to asset pricing puzzles. Let us take the equity premium puzzle as an example. The equity premium generated from the model is

$$E\left(R_2^e - R_1^f\right) = -R_1^f cov\left(\beta\frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})}, R_2^e\right), \quad \forall i, \tag{15}$$

or

$$E\left(R_2^e - R_1^f\right) = -R_1^f cov\left(\frac{1}{N}\sum_{i=1}^{N}\beta\frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})}, R_2^e\right). \tag{16}$$

The equity premium is positive because $U_{i2}^{'}(c_{i2})$ and R_2^e are negatively correlated for all $i \in \{1,\ldots,N\}$. Suppose that we model all the individuals using concave utility functions instead of the S-shaped utility functions. Then the covariance between $U_{i2}^{'}(c_{i2})$ and R_2^e becomes positive for all $i \in A$, and hence the right-hand side of Equation (15) is negative for these individuals. If we use Equation (16) to evaluate the model performance, the total equity premium generated by this model can be close to zero when the weighted sum of the negative risk premiums corresponding to the individuals in set A largely cancels out the weighted sum of the positive risk premiums associated with the individuals in set B.

In this simple two-state model, both risk averters and risk seekers demand a positive risk premium. Notice that an individual behaves as a risk seeker (resp. risk averter) when her consumption growth correlates negatively (resp. positively) with the risky return. For example, in Figure 1, the individual whose state-contingent plan is $(A|R_2^e = \underline{R}_2^e; E|R_2^e = \bar{R}_2^e)$ or $(B|R_2^e = \underline{R}_2^e; E|R_2^e = \bar{R}_2^e)$ is a risk averter, whereas the individual with the plan $(A|R_2^e = \underline{R}_2^e; F|R_2^e = \bar{R}_2^e)$ or $(B|R_2^e = \underline{R}_2^e; F|R_2^e = \bar{R}_2^e)$ is a risk seeker. It is easy to understand that a risk averter demands a positive risk premium. As the consumption growth of a risk averter correlates positively with the risky return, holding the risky asset would increase her consumption risks. However, for a risk seeker, the risky asset serves as insurance and plays a role of consumption smoothing because her consumption growth is negatively correlated with the risky return. As a result, risk-seeking individuals who prefer consumption risks to consumption certainty would also demand a positive risk premium of the insurance.

Besides, using concave utility instead of S-shaped utility accounts for the rejection of Hansen-Jagannathan inequality (1991). Hansen and Jagannathan (1991) give the following inequality

$$\sqrt{var\left[SDF_{t+1}\right]} \geq E\left(\frac{1}{R_t^f}\right)\frac{E(R_{t+1}^e - R_t^f)}{\sqrt{var(R_{t+1}^e - R_t^f)}}, \tag{17}$$

where SDF_{t+1} is the stochastic discount factor. Because the stochastic discount factor of a traditional C-CAPM generally has low volatility, this inequality is typically rejected. We decompose the stochastic discount factor $SDF = \frac{1}{N}\sum_{i=1}^N \beta \frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})}$ into two parts:

$$SDF = \frac{1}{N}\sum_{i\in A}\beta\frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})} + \frac{1}{N}\sum_{i\in B}\beta\frac{U_{i2}^{'}(c_{i2})}{U_{i1}^{'}(c_{i1})}. \tag{18}$$

Let SDF_A and SDF_B respectively denote the first and second terms in the right-hand side of equation (18). The variance of SDF can be written as

$$var(SDF) = var(SDF_A) + var(SDF_B) + 2cov(SDF_A, SDF_B).$$

In our model, $cov(SDF_A, SDF_B)$ is positive because the marginal utility $U_{i2}^{'}(c_{i2})$ is small (resp. large) at a good (resp. bad) state irrespective of whether individual i belongs to set A or set B. If we substitute the S-shaped utility functions with concave utility functions, then $cov(SDF_A, SDF_B)$ becomes negative. Actually, under the new utility functions, for all $i \in A$, the marginal utility $U_{i2}^{'}(c_{i2})$ at a good state becomes greater than that at a bad state. Apparently, the adoption of a globally concave utility function may lead to a small variance in the SDF.

4 Empirical evidence from micro consumption

This section presents empirical evidence from micro consumption data supporting the asset pricing model based on S-shaped consumption utility.

4.1 Testable prediction and identification

The S-shaped consumption utility predicts that some individuals' consumption growth correlates negatively with asset returns. The two-state model in Section 3 has shown that the risky return rate correlates negatively with risk seekers' consumption growth. Also, based on the opposite consumption responses among different individuals in this model, we infer that the risk-free rate in the real world is also negatively correlated with some individuals' consumption growth rates. Finding negative correlations would provide strong evidence supporting the S-shaped utility functions. At the least, we can use the evidence to argue that the consumption utility can not be globally concave-shaped. Otherwise, some demand a positive risk premium, but others command a negative one of the same risky return. The law of one price does not hold.[3]

However, identifying the partially negative correlation between consumption growth and asset returns faces serious challenges. First, we need to differentiate the individuals whose consumption growth is negatively correlated with asset returns from others for whom the correlation is positive. Second, even for one single individual, there are times when her consumption growth correlates negatively with asset returns and the times when the correlation is positive. It seems impractical to precisely differentiate the individuals and the timing. We need to propose an innovative approach to establish the existence of the partial negative correlation.

Our identification of the partial negative correlation relies on the heterogeneity of the consumption behavior at different consumption levels. To facilitate discussion, we classify the above consumption adjustments in Figure 1 into three categories: local moves, cycli-

[3]We can make this argument based on the identify

$$\begin{aligned} E_t[m_{t+1}R^e_{t+1}] &\equiv E_t m_{t+1} E_t R^e_{t+1} + cov_t(m_{t+1}, R^e_{t+1}) \\ &= \frac{1}{R^f_t} E_t R^e_{t+1} + cov_t(m_{t+1}, R^e_{t+1}), \end{aligned}$$

where m_{t+1} is the pricing kernel, and R^e_{t+1} and R^f_f are the equity return rate and the risk-free return rate, respectively. When $cov_t(m_{t+1}, R^e_{t+1})$ differs between two types of individuals, the prices ($E_t[m_{t+1}R^e_{t+1}]$) are different too.

cal jumps, and counter-cyclical jumps. The local moves refer to consumption adjustments within either the convex or concave section of the utility curve. As the concave section of the S-shaped utility function generally corresponds to high consumption level above a reference point, we refer to a point in this section as a high-consumption state. Similarly, we refer to a point in the convex section of the utility function as a low-consumption state. The local moves in the example (see the graph for period 2 in Figure 1) include the following changes: A to E, B to F, E to A, and F to B. The jumps refer to a large change of consumption from a high-consumption state to a low-consumption state or vice versa. If the jumps are positively related to the risky return rate, we say that they are cyclical jumps (e.g., B to E, and E to B). Otherwise, we call them counter-cyclical jumps (e.g., A to F, and F to A). We notice that consumption in low-consumption states is more likely to be negatively correlated with risky returns than consumption in high-consumption states. The reasons are two-fold. First and most importantly, local moves in low-consumption states correlate negatively with the risky returns, and the correlations in high consumption states are positive. Second, the counter-cyclical jumps (see the change from A to F or F to A in Figure 1) seem linked more to the low-consumption states than to the high-consumption states. It is worth mentioning that cyclical jumps may prevent us from observing the correlation differentiation between the low- and high-consumption states.

The above conclusion also applies to different consumption-growth states. Whether the reference point is different from the lagged consumption or not, a high (resp. low) consumption level more (resp. less) likely corresponds to a high (resp. low) consumption-growth rate. Therefore, we conjecture that consumption in low consumption-growth states more likely correlates negatively with risky returns than consumption in high consumption-growth states.

4.2 The econometric model

We perform a standard quantile regression analysis (Koenker and Bassett, 1978) to examine the correlations between asset return rates and different quantiles of consumption growth.

The quantiles correspond to the consumption-growth states discussed above. Hereafter, we use "consumption-growth quantiles" and "consumption-growth states" interchangeably. We adopt a quantile regression model of the following form:

$$log\left(\frac{c_{i,t+1}}{c_{i,t}}\right) = \delta_{0,\tau} + r_{t+1}\delta_{1,\tau} + u_{i,t+1}, \qquad (\tau \in (0,1);\ i = 1,\ldots,N_t;\ t = 1,\ldots,T), \quad (19)$$

where r_{t+1} is the logarithm of the real asset return rate (i.e., equity return rate R^e_{t+1} or risk-free return rate R^f_t); $u_{i,t+1}$ is an idiosyncratic error term; and N_t is the number of observations in period t. Two parameters, $\delta_{0,\tau}$ and $\delta_{1,\tau}$, are the intercept and the slope coefficient, respectively. We adopt the standard identification assumption that the τ^{th} quantile of the residual $u_{i,t+1}$ conditional on r_{t+1} is 0. To evaluate how asset returns vary with a wide range of consumption-growth states, we set $\tau = 0.05j$ for $j = 1,2,\ldots,19$.

The slope coefficient, $\delta_{1,\tau}$, links closely to the correlation between the τ^{th} quantile of consumption growth rate and asset returns. Denote by Q^{τ}_{t+1} the τ^{th} quantile of consumption growth rate, $log\left(\frac{c_{i,t+1}}{c_{i,t}}\right)$, between periods t and $t+1$. Ju and Li (2021) show that the ordinary least squares estimator $\hat{\delta}^{OLS}_{1,\tau}$ of $\delta_{1,\tau}$ in the following macroeconomic mean-regression model

$$Q^{\tau}_{t+1} = \delta_{0,\tau} + r_{t+1}\delta_{1,\tau} + u_{t+1}, \qquad (\tau \in (0,1);\ t = 1,\ldots,T), \quad (20)$$

is asymptotically equivalent to the standard quantile regression estimator of $\delta_{1,\tau}$ in model (19). Let σ_τ and σ_r be the standard deviations of Q^{τ}_{t+1} and r_{t+1}, respectively. It is easy to see that $\hat{\delta}^{OLS}_{1,\tau}$ consistently estimates $\frac{\sigma_\tau}{\sigma_r} corr(Q^{\tau}_{t+1}, r_{t+1})$, where *corr* denotes the correlation. Therefore, the quantile regression coefficient, $\delta_{1,\tau}$, is proportional to the correlation between the τ^{th} quantile of the consumption growth rate and asset returns. Throughout the paper, we interpret the slope coefficient, $\delta_{1,\tau}$, as a correlation.

In this paper, the merit of the quantile regression is to help us explore the existence of some individuals whose consumption growth correlates negatively with asset returns. Model (19), essentially a macro model, presents only cross-state (i.e., cross-quantile) heterogeneity in consumption behaviors. As suggested by Ju and Li (2021), the interpretation of the regression coefficient, $\delta_{1,\tau}$, needs to take into account the cross-individual heterogeneity, because the time-series evolution of the τ^{th} quantile of consumption growth over time is an outcome of heterogeneous changes of numerous individual consumption growth rates. In fact, the correlation between the τ^{th} quantile of consumption growth and asset returns differs significantly from the correlation between individual consumption growth rate and asset returns. We now clarify this point. Suppose that the status quo τ^{th} quantile of $log\left(\frac{c_{i,t+1}}{c_{i,t}}\right)$ is Q_{t+1}^{τ}. When the ex-ante state switches and asset returns rise, some individuals may increase their consumption while others reduce theirs. As the number of individuals whose consumption growth rates increase from a value below Q_{t+1}^{τ} to a value above Q_{t+1}^{τ} is smaller than the number of individuals whose growth rates decrease from a value above Q_{t+1}^{τ} to a value below Q_{t+1}^{τ}, the τ^{th} quantile of the new distribution for this period lies below Q_{t+1}^{τ}. Thus, Q_{t+1}^{τ} decreases. Therefore, if the estimated $\delta_{1,\tau}$ is significantly negative for some $\tau \in (0,1)$, it is evidence that some individuals' consumption growth rates are negatively correlated with asset returns.

It is possible that the estimated $\delta_{1,\tau}$ is positive for all $\tau \in (0,1)$. In this case, the number of individuals whose consumption growth is positively correlated with asset returns dominates that of individuals for whom the correlation is negative. However, we may find a negative correlation in some subgroups if this domination does not hold in these subsamples.

4.3 Data

The micro consumption data we use comes from the Consumption Expenditure Survey (CEX) conducted by the U.S. Bureau of Labor Statistics (BLS). We strictly follow the procedure of Ju and Li (2021) to process the consumption data. We aggregate hundreds of items of consumption expenditures into four classes: nondurable goods and services (NS); education and health expenditures (EH); durable goods (D); and other expenditures (O). According to different combinations of these classes, we define the following three types of consumption expenditure measures. Type I is the total household consumption expenditure. Type II includes only nondurable goods and services, a commonly used measure in many consumption-based models. Type III consists of what we call infrequent large-scale expenditures, including durable goods, education and health expenditures, and other expenditures. In formula, we have

- Type I $= \mathrm{NS} + \mathrm{EH} + \mathrm{D} + \mathrm{O}$,
- Type II $= \mathrm{NS}$,
- Type III $= \mathrm{D} + \mathrm{EH} + \mathrm{O}$.

Large-scale expenditures are generally not regarded as consumption. However, they are closely related to consumption and the change in consumption flow.

The primary data we use in this study are quarterly consumption growth rates. The final sample sizes of the five measures are 317,430 (Type I), 333,677 (Type II), 325,610 (Type III). The sample period spans 1982:Q2-2012:Q3 (excluding 1985:Q4).

The micro consumption differs greatly from the per capita consumption. For example, the volatility in micro consumption is much larger than that of per capita consumption. In our samples, the standard deviations of quarterly growth rates of consumption defined in Types I–III measures are 48.62%, 29.62%, 210.32%, respectively. In contrast, the standard deviation of per capita consumption growth rates[4] in 1982:Q2-2013:Q3 is merely 0.56%. Moreover, the micro consumption is much more volatile than the quarterly stock return rate. The standard deviation of the latter in the same period is only 6.47%.

Our measure of aggregate stock prices is the Standard & Poor's 500 Index. We compute quarterly dividends using Shiller's data (Shiller, 2000) which is available on his website. We calculate riskless return rates using the monthly return rates of 3-month Treasury bills. In our sample periods, the geometric means of the real quarterly return rates of stocks and Treasury bills are 2.03% and 0.35%, respectively. The annualized equity premium is 6.96%.

[4]The per capita consumption is calculated by dividing the deflated national expenditure of nondurable goods and services (provided by the Bureau of Economic Analysis) by the Civilian Noninstitutional Population (downloaded from the website of the Federal Reserve Bank of St. Louis).

Table 1: **Quantile regression (Model (19), Type-I consumption, whole sample)**

	$r_{t+1} = \log(R^e_{t+1})$					$r_{t+1} = \log(R^f_t)$				
	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$
$\tau =$	coef.	s.e.	coef.	s.e.	p-value	coef.	s.e.	coef.	s.e.	p-value
0.05	-0.6502	0.0033	-0.0144	0.0402	0.3840	-0.6408	0.0030	-2.7584	0.3667	0.0000
0.10	-0.4276	0.0020	-0.0340	0.0185	0.0320	-0.4230	0.0019	-1.6978	0.1954	0.0000
0.15	-0.3159	0.0013	-0.0308	0.0147	0.0120	-0.3128	0.0012	-1.1508	0.1539	0.0000
0.20	-0.2423	0.0011	-0.0297	0.0121	0.0080	-0.2405	0.0010	-0.7738	0.1259	0.0000
0.25	-0.1846	0.0008	-0.0142	0.0105	0.0760	-0.1840	0.0008	-0.3106	0.1050	0.0040
0.30	-0.1377	0.0008	-0.0154	0.0091	0.0640	-0.1380	0.0008	0.0140	0.0872	0.4160
0.35	-0.0965	0.0008	-0.0033	0.0090	0.3720	-0.0974	0.0008	0.3587	0.0860	0.0000
0.40	-0.0599	0.0007	0.0031	0.0083	0.3320	-0.0617	0.0007	0.6358	0.0776	0.0000
0.45	-0.0259	0.0007	0.0084	0.0080	0.1680	-0.0282	0.0007	0.8625	0.0896	0.0000
0.50	0.0074	0.0006	0.0130	0.0080	0.0640	0.0047	0.0006	1.1453	0.0823	0.0000
0.55	0.0411	0.0007	0.0167	0.0079	0.0240	0.0376	0.0007	1.4000	0.0781	0.0000
0.60	0.0761	0.0007	0.0191	0.0082	0.0080	0.0716	0.0007	1.6759	0.0733	0.0000
0.65	0.1141	0.0007	0.0302	0.0086	0.0000	0.1085	0.0007	1.9946	0.0881	0.0000
0.70	0.1550	0.0008	0.0383	0.0097	0.0000	0.1490	0.0008	2.2503	0.1027	0.0000
0.75	0.2038	0.0010	0.0429	0.0110	0.0000	0.1967	0.0010	2.6864	0.1082	0.0000
0.80	0.2634	0.0011	0.0605	0.0131	0.0000	0.2545	0.0011	3.2231	0.1222	0.0000
0.85	0.3420	0.0014	0.0598	0.0137	0.0000	0.3300	0.0014	3.7836	0.1459	0.0000
0.90	0.4579	0.0019	0.0808	0.0220	0.0040	0.4439	0.0018	4.6289	0.2000	0.0000
0.95	0.6798	0.0033	0.0830	0.0333	0.0040	0.6613	0.0033	5.3289	0.3464	0.0000

4.4 Estimation results

Tables 1 – 4 present the estimation results of quantile regressions of different consumption growth rates on asset return rates. In all these tables, we report not only the standard errors of the estimates but also the p-values of testing the significance of their signs. All estimates, standard errors, and p-values are computed based on bootstrapping using sampling with replacement (replicated 250 times). We denote $\hat{\delta}_{0,\tau}$ and $\hat{\delta}_{1,\tau}$ as the estimates of $\delta_{0,\tau}$ and $\delta_{1,\tau}$, respectively. In all four tables, the estimates of the intercepts $\delta_{0,\tau}$ do not show anything unusual. Each estimate $\hat{\delta}_{0,\tau}$ is around the τ^{th} quantile of the corresponding distribution of consumption growth rates. All our interesting findings are from the estimates of $\delta_{1,\tau}$, which we summarize as follows.

First, the estimation results show that the consumption growth (measures of Type I and Type III) of many individuals in the whole population varies negatively with asset returns. In Tables 1 and 2, we observe that many estimates of $\delta_{1,\tau}$ at low quantiles ($\tau < 0.5$) are negative and statistically significant at the 5% or 1% level (in a one-sided test context). All

Table 2: **Quantile regression (Model (19), Type-III consumption, whole sample)**

	$r_{t+1} = \log(R^e_{t+1})$					$r_{t+1} = \log(R^f_t)$				
	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$
$\tau =$	coef.	s.e.	coef.	s.e.	p-value	coef.	s.e.	coef.	s.e.	p-value
0.05	-1.5879	0.0065	-0.1533	0.0885	0.0680	-1.5646	0.0073	-7.7096	0.8502	0.0000
0.10	-1.0540	0.0051	-0.1813	0.0581	0.0040	-1.0365	0.0046	-6.3820	0.5104	0.0000
0.15	-0.7421	0.0036	-0.1921	0.0402	0.0000	-0.7294	0.0033	-5.1713	0.3625	0.0000
0.20	-0.5431	0.0022	-0.1518	0.0278	0.0000	-0.5345	0.0023	-3.6276	0.3124	0.0000
0.25	-0.3955	0.0022	-0.0992	0.0262	0.0000	-0.3911	0.0021	-2.3725	0.2465	0.0000
0.30	-0.2839	0.0019	-0.0776	0.0242	0.0000	-0.2822	0.0019	-1.2809	0.2203	0.0000
0.35	-0.1931	0.0016	-0.0637	0.0180	0.0000	-0.1931	0.0016	-0.3659	0.1909	0.0200
0.40	-0.1174	0.0014	-0.0397	0.0162	0.0120	-0.1192	0.0014	0.4416	0.1527	0.0040
0.45	-0.0532	0.0013	-0.0156	0.0132	0.1240	-0.0559	0.0012	1.0455	0.1445	0.0000
0.50	-0.0019	0.0009	0.0072	0.0096	0.2520	-0.0042	0.0010	1.5567	0.1968	0.0000
0.55	0.0550	0.0012	0.0257	0.0156	0.0400	0.0490	0.0011	2.6907	0.1445	0.0000
0.60	0.1211	0.0014	0.0580	0.0149	0.0000	0.1125	0.0013	3.6835	0.1565	0.0000
0.65	0.1978	0.0016	0.0820	0.0200	0.0000	0.1862	0.0015	4.6456	0.1565	0.0000
0.70	0.2878	0.0019	0.1175	0.0195	0.0000	0.2728	0.0020	5.6922	0.2125	0.0000
0.75	0.4007	0.0020	0.1342	0.0248	0.0000	0.3826	0.0020	6.9683	0.2628	0.0000
0.80	0.5441	0.0027	0.1490	0.0281	0.0000	0.5195	0.0025	8.2343	0.3092	0.0000
0.85	0.7432	0.0038	0.1517	0.0426	0.0000	0.7142	0.0033	9.4509	0.4041	0.0000
0.90	1.0522	0.0042	0.1535	0.0510	0.0000	1.0149	0.0046	10.5050	0.5472	0.0000
0.95	1.5830	0.0059	0.1177	0.0728	0.0440	1.5482	0.0059	9.5332	0.7808	0.0000

Table 3: **Quantile regression (Model (19), Type-II consumption, whole sample)**

	$r_{t+1} = \log(R^e_{t+1})$					$r_{t+1} = \log(R^f_t)$				
	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$
$\tau =$	coef.	s.e.	coef.	s.e.	p-value	coef.	s.e.	coef.	s.e.	p-value
0.05	-0.4335	0.0013	0.0154	0.0153	0.1400	-0.4337	0.0013	0.1456	0.1490	0.1520
0.10	-0.3198	0.0011	0.0101	0.0113	0.2000	-0.3201	0.0010	0.1583	0.1131	0.0760
0.15	-0.2480	0.0008	0.0008	0.0090	0.4640	-0.2486	0.0008	0.1906	0.1117	0.0440
0.20	-0.1937	0.0007	-0.0067	0.0092	0.2440	-0.1945	0.0007	0.2214	0.0922	0.0000
0.25	-0.1506	0.0007	-0.0004	0.0082	0.4800	-0.1515	0.0007	0.3031	0.0895	0.0000
0.30	-0.1125	0.0007	0.0013	0.0080	0.4320	-0.1138	0.0007	0.4064	0.0780	0.0000
0.35	-0.0792	0.0006	-0.0037	0.0074	0.3160	-0.0807	0.0006	0.4712	0.0691	0.0000
0.40	-0.0477	0.0006	-0.0012	0.0069	0.4320	-0.0495	0.0006	0.6179	0.0688	0.0000
0.45	-0.0181	0.0006	0.0068	0.0061	0.1160	-0.0203	0.0006	0.7477	0.0728	0.0000
0.50	0.0107	0.0006	0.0091	0.0057	0.0520	0.0082	0.0006	0.8642	0.0720	0.0000
0.55	0.0401	0.0006	0.0145	0.0068	0.0160	0.0375	0.0006	0.9844	0.0701	0.0000
0.60	0.0705	0.0006	0.0182	0.0066	0.0000	0.0674	0.0006	1.1510	0.0690	0.0000
0.65	0.1026	0.0007	0.0234	0.0074	0.0000	0.0989	0.0007	1.3251	0.0727	0.0000
0.70	0.1379	0.0007	0.0195	0.0081	0.0120	0.1335	0.0007	1.5073	0.0718	0.0000
0.75	0.1776	0.0007	0.0238	0.0078	0.0000	0.1722	0.0006	1.7330	0.0818	0.0000
0.80	0.2241	0.0010	0.0286	0.0093	0.0000	0.2185	0.0009	1.9919	0.0946	0.0000
0.85	0.2818	0.0010	0.0267	0.0105	0.0000	0.2745	0.0009	2.3400	0.1014	0.0000
0.90	0.3583	0.0011	0.0387	0.0129	0.0000	0.3498	0.0010	2.7456	0.1290	0.0000
0.95	0.4827	0.0016	0.0345	0.0166	0.0200	0.4720	0.0015	3.1578	0.1740	0.0000

Table 4: **Quantile regression (Model (19), Type-II consumption, young households)**

	$r_{t+1} = \log(R^e_{t+1})$					$r_{t+1} = \log(R^f_t)$				
	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$	$\delta_{0,\tau}$		$\delta_{1,\tau}$		$sign(\delta_{1,\tau})$
$\tau =$	coef.	s.e.	coef.	s.e.	p-value	coef.	s.e.	coef.	s.e.	p-value
0.05	-0.4355	0.0035	-0.0622	0.0406	0.0720	-0.4337	0.0033	-0.9255	0.3707	0.0120
0.10	-0.3167	0.0028	-0.0287	0.0330	0.1840	-0.3138	0.0031	-0.7877	0.3258	0.0000
0.15	-0.2447	0.0024	-0.0623	0.0255	0.0120	-0.2422	0.0024	-0.8887	0.2721	0.0000
0.20	-0.1917	0.0020	-0.0495	0.0195	0.0000	-0.1893	0.0019	-0.8180	0.2348	0.0000
0.25	-0.1484	0.0017	-0.0414	0.0189	0.0120	-0.1474	0.0016	-0.6172	0.2070	0.0000
0.30	-0.1115	0.0016	-0.0440	0.0200	0.0160	-0.1117	0.0019	-0.2308	0.1721	0.0800
0.35	-0.0791	0.0014	-0.0407	0.0190	0.0000	-0.0795	0.0016	-0.1275	0.1797	0.2240
0.40	-0.0482	0.0016	-0.0416	0.0187	0.0160	-0.0490	0.0015	-0.0076	0.1680	0.4480
0.45	-0.0189	0.0015	-0.0229	0.0152	0.0600	-0.0202	0.0014	0.1799	0.1505	0.1160
0.50	0.0089	0.0013	-0.0127	0.0172	0.2240	0.0079	0.0013	0.2460	0.1531	0.0400
0.55	0.0371	0.0014	-0.0065	0.0149	0.3440	0.0355	0.0015	0.4188	0.1871	0.0120
0.60	0.0667	0.0017	0.0034	0.0173	0.4640	0.0646	0.0016	0.6290	0.1847	0.0000
0.65	0.0977	0.0016	0.0077	0.0187	0.3160	0.0947	0.0018	0.8936	0.1861	0.0000
0.70	0.1317	0.0017	0.0029	0.0188	0.4360	0.1283	0.0018	1.1170	0.1993	0.0000
0.75	0.1726	0.0020	0.0268	0.0257	0.1560	0.1681	0.0019	1.4500	0.2172	0.0000
0.80	0.2194	0.0021	0.0320	0.0265	0.1000	0.2144	0.0021	1.5102	0.2538	0.0000
0.85	0.2742	0.0023	0.0394	0.0254	0.0640	0.2678	0.0022	1.6690	0.2326	0.0000
0.90	0.3493	0.0026	0.0477	0.0321	0.0480	0.3430	0.0028	1.8079	0.3839	0.0000
0.95	0.4745	0.0037	0.0713	0.0397	0.0360	0.4682	0.0044	1.9738	0.5135	0.0000

estimates at top quantiles ($\tau > 0.5$) (in Tables 1 and 2) are significantly positive. The cross-quantile pattern of the estimates of $\delta_{1,\tau}$ indicates that the distribution of micro consumption growth tends to "expand" when asset returns rise. We call this phenomenon "big bang," a term coined from Ju and Li (2021).

Second, for Type-II consumption, we find no direct evidence of the partial negative correlation for the whole population but find direct evidence within a subgroup, i.e., young households. In Table 3, we observe that the low-quantile estimates of $\delta_{1,\tau}$ in both the left and the right panels are close to zero, indicating a low correlation between asset returns and low quantiles of growth rates of nondurable goods and services. However, the top-quantile estimates of $\delta_{1,\tau}$ are much larger. The failure of finding the direct evidence does not mean there are no individuals whose Type-II consumption growth correlates negatively with asset returns. We substantiate this conjecture with a subgroup analysis. Table 4 reports the estimates of the quantile regression model (19) for young households (household heads aged less than or equal to 30). We observe that the correlations between stock return (or risk-free return) and many low quantiles of Type-II consumption growth rates of the young subgroup are significantly negative, which is strikingly different from the findings in the whole sample (see Table 3).

Third, the quantile regression offers a finding that we call "accelerated expansion." We interpret the regression coefficient as the "velocity" at which the particular quantile moves in response to a rise in asset returns. We observe that, in Tables 1, 2, and 4, the greater the distance of a quantile from the "big bang center" where the velocity is zero, the faster it moves away. After the structural estimation in section 5, we present an explanation for the accelerated expansion.

We also find a partial negative correlation in stockholders' Type-II consumption. Appendix A reports a salient big bang in Type-II consumption of some subgroups of stockholders. By dividing the stockholders or young stockholders into subgroups according to one-period lagged consumption growth, we find that the low quantiles of Type-II consumption growth rates of several subgroups vary negatively with stock returns or risk-free returns. Refer to Tables A.1 and A.2.

4.5 Ju and Li's (2021) evidence

Ju and Li (2021) find a partial negative correlation in some subgroups for which this paper fails to present direct evidence. Ju and Li (2021) estimate the following quantile regression model:

$$Q_{t+1}^{\tau} = \delta_{0,\tau} + r_{t+1}\delta_{1,\tau} + \lambda_{\tau}^{'} F_{t+1} + u_{t+1}, \qquad (\tau \in (0,1);\ t = 1,\ldots,T), \tag{21}$$

where Q_{t+1}^{τ} is defined the same as in (20) except that the consumption is of Type-II measure; r_{t+1} is the logarithm of the gross real interest rate which is known in period t; F_{t+1} is a vector of common factors representing the state of the economy. Ju and Li (2021) use the first seven principal components of hundreds of U.S. macroeconomic time series as the common factors. By computing the correlation, $corr(\lambda_{\tau_1}^{'} F_{t+1}, \lambda_{\tau_2}^{'} F_{t+1})$, they show that, for many groups, the responses of the top quantiles ($\tau_2 > 0.5$) of consumption growth to changes in common factors are generally oppositive to those of the low quantiles ($\tau_1 < 0.5$). These groups include the whole population, young households, prime-age households (household heads aged more than 30 but less than or equal to 60), old households (household heads aged more than 60), and non-stockholders.

Ju and Li (2021) do not find a negative correlation, $corr(\lambda_{\tau_1}^{'}F_{t+1}, \lambda_{\tau_2}^{'}F_{t+1})$ ($\tau_1 < 0.5$ and $\tau_2 > 0.5$) for stockholders. They argue that the wealth effects of changes in stock prices induce stockholders more likely to have cyclical consumption jumps than non-stockholders; therefore, it is much harder to detect the negative correlation in the sample of stockholders. However, Ju and Li (2021) present some indirect evidence using a filtering method.

4.6 Robustness against measurement errors in consumption

It is well-known that survey data contains a large amount of noise. In linear mean regression, classic measurement errors in the dependent variable does not affect unbiasedness. However, Hausman et al. (2019) and Ju and Li (2021) show that this type of error might cause the linear quantile regression estimator of Koenker and Bassett (1978) to be biased.

In Web Appendix B, we propose a conditional deconvolution method to deal with the measurement errors in consumption. Overall, we find that the phenomena of the big bang and accelerated expansion are quite robust against measurement errors. All of the qualitative conclusions remain unchanged.

5 A structural model estimation

With an estimable structural asset pricing model based on micro consumption and reference-dependent preference, this section shows that an S-shaped consumption utility can quantitatively solve the equity premium puzzle.

5.1 The structural model

5.1.1 The consumer's optimization problem

Consider an economy in which there are N consumers indexed by $i = 1, \ldots, N$. The i^{th} consumer maximizes her lifetime utility

$$E\left[\sum_{\tau=0}^{\infty} \delta^{\tau} U_{i,t+\tau} | I_{it}\right], \tag{22}$$

where $U_{i,t+\tau}$ is her intraperiod consumption utility, I_{it} is her information set at time t, and δ measures the subjective time discount factor. We specify $U_{i,t+\tau}$ as a reference-dependent utility function

$$U_{i,t+\tau} = \mu\left(\ln\left(\frac{c_{i,t+\tau}}{\theta_{i,t+\tau}}\right)\right), \tag{23}$$

where μ is a gain-loss utility function, $c_{i,t+\tau}$ denotes the consumption expenditure on non-durable goods and services, and $\theta_{i,t+\tau}$ represents the reference point, which is individual specific and time-varying. The argument of the gain-loss function is the consumption-reference ratio (in logarithm). Individuals perceive the ratios as gains if they are greater than or equal to zero and losses otherwise. For ease of exhibition, decompose $\mu(\cdot)$ into three parts:

$$\mu\left(\ln\left(\frac{c_{i,t+\tau}}{\theta_{i,t+\tau}}\right)\right) = \begin{cases} \mu_g\left(\ln\left(\frac{c_{i,t+\tau}}{\theta_{i,t+\tau}}\right)\right), & \text{if } c_{i,t+\tau} > \theta_{i,t+\tau}; \\ 0, & \text{if } c_{i,t+\tau} = \theta_{i,t+\tau}; \\ \mu_l\left(\ln\left(\frac{c_{i,t+\tau}}{\theta_{i,t+\tau}}\right)\right), & \text{if } c_{i,t+\tau} < \theta_{i,t+\tau}, \end{cases} \tag{24}$$

where $\mu_g(\cdot)$ and $\mu_l(\cdot)$ denote the utility functions for the gains and losses, respectively. Following Kahneman and Tversky (1979), we impose the following restrictions on the gain-loss function $\mu(\cdot)$: (i) $\mu_g(0) = \mu_l(0) = 0$, (ii) $\mu_g^{'}(z) \geq 0$ for all $z > 0$, (iii) $\mu_l^{'}(z) \geq 0$ for all $z < 0$, (iv) $\mu_g^{''}(z) \leq 0$ for all $z > 0$, (v) $\mu_l^{''}(z) \geq 0$ for all $z < 0$. Condition (i) is a standard normalization. Conditions (ii) and (iii) restrict $\mu(\cdot)$ to be monotonically increasing. The last two conditions guarantee that $\mu(\cdot)$ is an S-shaped consumption utility function; that is, $\mu(c_{i,t+\tau})$ is concave if $c_{i,t+\tau} > \theta_{i,t+\tau}$ and convex if $c_{i,t+\tau} < \theta_{i,t+\tau}$. We assume that the reference point $\theta_{i,t+\tau}$ is exogenous and specify it later in Section 5.1.2.[5]

There are $K+1$ tradable financial assets in the economy indexed by $k = 0,1,\ldots,K$. The 0^{th} asset denotes a risk-free security, and the remaining ones are risky assets. We represent the return rate of the k^{th} asset in period t as $R_{k,t}(k = 0,\ldots,K)$. We specify the i^{th} consumer's intertemporal budget constraint as

$$c_{it} + \sum_{k=0}^{K} a_{ik,t} = \sum_{k=0}^{K} a_{ik,t-1} R_{k,t} + Y_{it}, \tag{25}$$

where $a_{ik,t}$ is her investment in security k in period t, and Y_{it} is her labor income.

As the intertemporal preference is time separable, the shadow price of wealth, λ_{it}, for the i^{th} consumer in period t is the marginal utility of consumption within the period; that is,

$$\lambda_{it} = \mu^{'}\left(\ln(\frac{c_{i,t}}{\theta_{i,t}})\right)\frac{1}{c_{it}}. \tag{26}$$

Note that the marginal utility at the reference points may not be well defined. Throughout the paper, we assume zero probability that consumption choices fall on the reference points. Given the shadow price formula, the SDF or pricing kernel for the i^{th} consumer is

$$M_{i,t+1} = \delta\frac{\lambda_{i,t+1}}{\lambda_{it}} = \frac{\mu^{'}(\ln(\frac{c_{i,t+1}}{\theta_{i,t+1}}))c_{i,t}}{\mu^{'}(\ln(\frac{c_{i,t}}{\theta_{it}}))c_{i,t+1}}. \tag{27}$$

The first-order conditions for this individual's optimization imply the Euler equation:

$$E\left[M_{i,t+1}R_{k,t+1}|I_{it}\right] = 1, \tag{28}$$

for all assets $k = 0,1,\ldots,K$.

[5]Unlike Kahneman and Tversky (1979), we do not restrict the consumers to be loss averse. Loss aversion implies that individuals are more sensitive to losses than to the same amount of gains. Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001) show that loss aversion has significant ramifications for asset pricing. In our model, whether individuals are loss averse or not is determined by a data-driven approach.

5.1.2 Specification of reference points

While the reference points are central in prospect theory, Kahneman and Tversky (1979) offer relatively little guidance on how they are determined. Barberis (2013) points out that the lack of a general rule for choosing reference points prevents researchers from developing applications of prospect theory.

We specify the reference point as a distribution closely related to the lagged consumption

$$\theta_{it} = c_{i,t-1}\theta_i^t,$$

where θ_i^t is a stationary random variable. In our data set, not many individuals report more than three periods of consumption expenditures. Therefore, we further restrict that

$$\theta_{it} = c_{i,t-1}\theta_i^1, \quad \theta_{i,t+1} = c_{i,t}\theta_i^2. \tag{29}$$

Denote $g_{it} = \frac{c_{i,t+1}}{c_{it}}$ as the i^{th} consumer's growth rate of consumption from t to $t+1$. Assume that the two latent reference points given the observed consumption growth rates, g_{it} and $g_{i,t+1}$, follow a joint log-normal distribution:

$$\left(\begin{array}{c} \ln(\theta_i^1) \\ \ln(\theta_i^2) \end{array} \middle| (g_{i,t}, g_{i,t+1}) \right) \sim N \left[\left(\begin{array}{c} r^1(g_{it}, g_{i,t+1}) \\ r^2(g_{it}, g_{i,t+1}) \end{array} \right), \left(\begin{array}{cc} \sigma_r^2 & \rho\sigma_r^2 \\ \rho\sigma_r^2 & \sigma_r^2 \end{array} \right) \right], \tag{30}$$

where $r^j(g_{it}, g_{i,t+1}), j = 1,2$ denote the conditional means, and σ_r^2 and ρ denote the variance and the correlation, respectively.[6] We specify $r^j(g_{it}, g_{i,t+1})$ as a translog function:

$$r^j(g_{it}, g_{i,t+1}) = r_1^j \log(g_{it}) + r_2^j \log(g_{i,t+1}), \quad j = 1,2. \tag{31}$$

[6]Note that we implicitly assume that the reference points are conditionally independent of $(R_{0,t+1}, \ldots, R_{K,t+1})$. Therefore, the pricing kernel of our model contains no information about prices. In the existing literature, this information improves model performance in explaining the cross-section stock returns (e.g., Fama and French, 1993, 2015; Bansal, Dittmar, Lundblad, 2005). However, the inclusion of price information in the pricing kernel makes the model not purely consumption-based, distorting our understanding of individuals' preferences toward consumption risks.

5.1.3 The aggregate pricing kernel

Given the above specifications of reference points, the pricing kernel in Equation (28) becomes

$$M_{i,t+1} \equiv M(\theta_i^1, \theta_i^2, g_{it}, g_{i,t+1}) = \delta \frac{\mu^{'}(\ln(\frac{g_{i,t+1}}{\theta_i^2}))g_{i,t+1}^{-1}}{\mu^{'}(\ln(\frac{g_{i,t}}{\theta_i^1}))}. \tag{32}$$

Let I_t be the set of common information; that is, $I_t = \bigcap_{i=1}^N I_{it}$. Using the tower property of conditional expectation, we rewrite the asset pricing equations (28) as

$$E\left[M(\theta_i^1, \theta_i^2, g_{it}, g_{i,t+1})R_{k,t+1}|I_t, g_{it}\right] = 1, \quad k = 0, 1, \ldots, K. \tag{33}$$

Let Θ represent all parameters in our model. Define a conditional expectation $m(g_{it}, g_{i,t+1}, \Theta)$ as follows:

$$m(g_{it}, g_{i,t+1}, \Theta) = E\left[M(\theta_i^1, \theta_i^2, g_{it}, g_{i,t+1})\right| (g_{it}, g_{i,t+1}, I_t, R_{0,t+1}, \ldots, R_{K,t+1})\big]. \tag{34}$$

As the specification (30) indicates that the reference points are conditionally independent of $(I_t, R_{0,t+1}, \ldots, R_{K,t+1})$, we therefore compute the integral in (34) by

$$m(g_{it}, g_{i,t+1}, \Theta) = \iint M(\theta_i^1, \theta_i^2, g_{it}, g_{i,t+1}) f(\theta_i^1, \theta_i^2|(g_{it}, g_{i,t+1}))d\theta_i^1 d\theta_i^2, \tag{35}$$

where f denotes the conditional density function. Combining Equations (33) and (34) and applying the tower property of conditional expectation twice, we have

$$E\left[m(g_{it}, g_{i,t+1}, \Theta)R_{k,t+1}|I_t\right] = 1, \quad k = 0, 1, \ldots, K. \tag{36}$$

To alleviate the issue of measurement errors in consumption data, we take an average of (36) across all individuals, obtaining the aggregate pricing kernel

$$\bar{m}_{t+1}(\Theta) = \frac{1}{N}\sum_{i=1}^N m(g_{it}, g_{i,t+1}, \Theta), \tag{37}$$

which satisfies the following asset pricing equation:

$$E\left[\bar{m}_{t+1}(\Theta)R_{k,t+1}\middle|I_t\right] = 1, \quad k = 0, 1, \ldots, K. \tag{38}$$

5.1.4 Specification of the gain-loss utility function

We specify the utility functions for the gains and losses respectively as

$$\mu_g(z) = \sum_{\nu=0}^{n} \beta_\nu^g b_{\nu,n}^{[0,\bar{z}]}(z) \tag{39}$$

and

$$\mu_l(z) = \sum_{\nu=0}^{n} \beta_\nu^l b_{\nu,n}^{[\underline{z},0]}(z), \tag{40}$$

where β_ν^g and β_ν^l are coefficients, $[\underline{z}, \bar{z}]$ is the domain of the gain-loss utility function μ, and

$$b_{\nu,n}^{[a,b]}(z) = \frac{1}{(b-a)^n}\binom{n}{\nu}(z-a)^\nu(b-z)^{n-\nu}, \quad \nu = 0, \ldots, n, \tag{41}$$

are Bernstein basis polynomials. To have an S-shaped gain-loss utility, we impose the restrictions on the coefficients as follows: (I) $\beta_0^g = \beta_n^l = 0$, (II) $\beta_\nu^g \leq \beta_{\nu+1}^g, \nu = 0, \ldots, n-1$, (III) $\beta_\nu^l \leq \beta_{\nu+1}^l, \nu = 0, \ldots, n-1$, (IV) $\beta_{\nu+2}^g - 2\beta_{\nu+1}^g + \beta_\nu^g \leq 0, \nu = 0, \ldots, n-2$, (V) $\beta_{\nu+2}^l - 2\beta_{\nu+1}^l + \beta_\nu^l \geq 0, \nu = 0, \ldots, n-2$.[7] Conditions (I)–(V) are sufficient for (i)–(v) in section 5.1.1 to hold. Notice that all Bernstein coefficients are identifiable only up to a multiplier; we therefore impose a normalization condition: $\beta_1^g = 1$.

[7]The first-order derivative of a Bernstein polynomial of degree n $B_n(z) = \sum_{\nu=0}^{n} \beta_\nu b_{\nu,n}^{[a,b]}(z)$ is $B_n'(z) = n\sum_{\nu=0}^{n-1}(\beta_{\nu+1} - \beta_\nu)b_{\nu,n-1}^{[a,b]}(z)$. Its second-order derivative is $B_n''(z) = n(n-1)\sum_{\nu=0}^{n-2}(\beta_{\nu+2} - 2\beta_{\nu+1} + \beta_\nu)b_{\nu,n-2}^{[a,b]}(z)$. Restricting the coefficients in the first-order and second-order derivatives according to conditions (II)-(V) guarantees that the gain-loss utility function is monotonically increasing and S-shaped.

5.2 Econometric estimation and nonregularity

From (38), we have

$$E\left[\bar{m}_{t+1}(\Theta)R_{0,t+1}\right] = 1 \tag{42}$$

and

$$E\left[\bar{m}_{t+1}(\Theta)(R_{k,t+1} - R_{0,t+1})\right] = 0, \quad k = 1, \ldots, K. \tag{43}$$

We can identify the preference parameters from the unconditional moment conditions (42) and (43) when the number of test assets, $K+1$, is no less than the number of parameters. Denote $\mathbb{R}_{t+1} = (R_{0,t+1}, R_{1,t+1} - R_{0,t+1}, \ldots, R_{K,t+1} - R_{0,t+1})'$ as the payoff vector of the portfolios with a price vector $p = (1, 0, \ldots, 0)'$. The vector of pricing errors is

$$\Phi_T(\Theta) = \frac{1}{T}\sum_{t=1}^{T}\left[\bar{m}_{t+1}(\Theta)\mathbb{R}_{t+1} - p\right]. \tag{44}$$

We minimize the distance

$$d(\Theta) = \sqrt{\Phi_T'(\Theta)W_K\Phi_T(\Theta)}, \tag{45}$$

where the weight matrix is

$$W_K = \begin{pmatrix} K & 0 \\ 0 & I_K \end{pmatrix}.$$

In the following, we call $d(\Theta)$ the WK distance. The designed block-diagonal weight matrix W_K assigns equal weights to the squared pricing error of the risk-free return and the (cross-portfolio) average squared pricing error of excess returns.

It is worth mentioning that it is difficult to carry out statistical inference. The econometric model is nonregular as we allow the reference point to be a kink point of the utility curve. Given an arbitrarily small change in, say, σ_r^2, there is a nonzero mass of individuals whose marginal utility jumps. Thus, the pricing kernels of these individuals, and hence the aggregate pricing kernel, are not differentiable with respect to σ_r^2. As a consequence, the

WK distance $d(\Theta)$ is generally nondifferentiable. It is thus infeasible to derive the limiting distribution of $\hat{\Theta}$ by following traditional methods of moments.

5.3 Estimation and results

5.3.1 Estimation

Several preparations are needed to implement the estimation. First, a full specification of the gain-loss utility function is required. In our dataset, the standard deviation of consumption growth (in logarithm) is 0.2874, and the minimum and maximum values are -0.9470 and 0.9974, respectively. However, the consumption-reference ratios (in logarithms) may go beyond the range $[-0.9479, 0.9974]$. For this reason, we specify the domains of the gain function $\mu_g(z)$ and the loss function $\mu_l(z)$ as $[0, 1.5]$ and $[-1.5, 0]$, respectively. Thus, the domain of μ is over ten times as wide as the standard deviation of consumption growth. In addition, to have flexible specifications of $\mu_g(z)$ and $\mu_l(z)$, we fix the order of the two Bernstein polynomials to eight. Second, we impose several meaningful restrictions on the estimate parameters. For example, we restrict the subjective time discount factor δ to the range $[0.9000, 0.9999]$, the conditional variance of reference points σ_r^2 to the range $(0, \infty)$, and the correlation ρ to the range $(-1, 1)$. Third, we choose Fama and French's 100 portfolios sorted by size and book-to-market ratio as test assets. Among them, four portfolios encounter the problem of missing data and are dropped. In addition, we add the three-month Treasury bill as a test asset. Therefore, the number of moment conditions in the estimation is 97. Fourth, we choose stockholders' nondurable goods and services as the measure of consumption.

We compute the double integral in Equation (35) using a simulated method. Given the specifications in (30) and (31), it is easy to draw observations randomly from the conditional distribution of the reference points. Suppose we repeat drawing R times. Let $(\theta_{i,r}^1, \theta_{i,r}^2)$

denote the r^{th} repetition. According to the law of large numbers, the simulated average

$$\frac{1}{R}\sum_{r=1}^{R} M(\theta_{i,r}^{1}, \theta_{i,r}^{2}, g_{it}, g_{i,t+1}) \tag{46}$$

is a consistent estimator of $m(g_{it}, g_{i,t+1}, \Theta)$. Therefore, we can compute the complicated double integral in Equation (35) using the simple simulated average.[8]

5.3.2 Model performance

With estimated preference parameters, we can define the fitted expected return rates and fitted risk premiums. Denote $\hat{\Theta}$ the estimate of Θ. Then, $\bar{m}_{t+1}(\hat{\Theta})$ is the estimated aggregate pricing kernel. From the identities

$$cov[\bar{m}_{t+1}(\hat{\Theta}), R_{k,t+1}] = E[\bar{m}_{t+1}(\hat{\Theta})R_{k,t+1}] - E[\bar{m}_{t+1}(\hat{\Theta})]ER_{k,t+1}, \quad k = 0, 1, \ldots, K,$$

we have the formula of expected returns:

$$ER_{k,t+1} = \frac{E[\bar{m}_{t+1}(\hat{\Theta})R_{k,t+1}] - cov[\bar{m}_{t+1}(\hat{\Theta}), R_{k,t+1}]}{E[\bar{m}_{t+1}(\hat{\Theta})]}, \quad k = 0, 1, \ldots, K.$$

We define the fitted expected returns as

$$\hat{\hat{R}}_{k,t+1} = \frac{1 - \widehat{cov}[\bar{m}_{t+1}(\hat{\Theta}), R_{k,t+1}]}{\hat{E}[\bar{m}_{t+1}(\hat{\Theta})]}, \quad k = 0, 1, \ldots, K, \tag{47}$$

where $\hat{E}$ and $\widehat{cov}$ indicate the sample mean and sample covariance, respectively. If $\bar{m}_{t+1}(\hat{\Theta})$ prices the kth asset correctly—that is, $E[\bar{m}_{t+1}(\hat{\Theta})R_{k,t+1}] = 1$—then the fitted expected return $\hat{\hat{R}}_{k,t+1}$ is asymptotically equal to the expected return $ER_{k,t+1}$. Otherwise, there is a difference between $\hat{\hat{R}}_{k,t+1}$ and $ER_{k,t+1}$. Similarly, we define the fitted market risk premium

[8] As the number of individuals in our sample is considerably large (27,914), we set R to 100. In addition, minimizing the WK distance $d(\Theta)$ defined in (45) requires a multi-start or global algorithm. Local optimization typically converges very slowly, presumably because the objective function $d(\Theta)$ is not differentiable with respect to certain parameters. We run MATLAB's global optimization procedure (Matlab 2020b) multiple times with different starting solutions.

as

$$\frac{-\widehat{cov}[\bar{m}_{t+1}(\hat{\Theta}), R_{t+1} - R_{0,t+1}]}{\hat{E}[\bar{m}_{t+1}(\hat{\Theta})]}, \tag{48}$$

where R_{t+1} is the market equity return. If $\bar{m}_{t+1}(\hat{\Theta})$ prices the market excess return $R_{t+1} - R_{0,t+1}$ correctly—that is, $E[\bar{m}_{t+1}(\hat{\Theta})(R_{t+1} - R_{0,t+1})] = 0$—then the fitted market risk premium is asymptotically equal to the genuine market risk premium $E(R_{t+1} - R_{0,t+1})$.

Figure 2 plots the fitted expected returns against the mean realized returns. All of the circles in the figure denote Fama and French's 96 portfolios sorted by size and book-to-market ratio. Two solid black points represent the risk-free return and stock market return, respectively. If the model fits the data perfectly, all of the points and circles in the figure will lie along the 45-degree line.

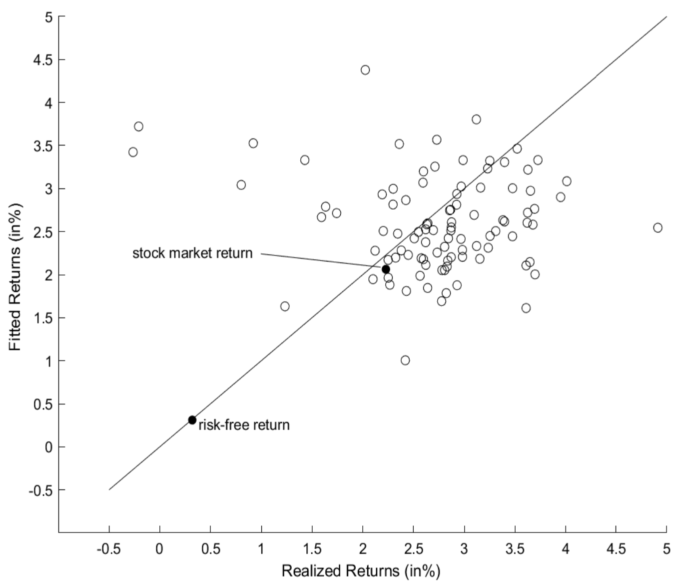

Figure 2: Realized vs. Fitted Returns

The model has some explanatory power on the cross-section expected stock returns. Its performance is comparable to that of CAPM as the computed WK distance is 0.1003 while the distance estimated for CAPM is 0.1009. However, this model fits the data less well compared with the Fama and French's three-factor model because the latter achieves a much smaller WK distance (0.0712). More work is needed to study the cross-section performance of an asset pricing model based on S-shaped consumption utility. As this is not the focus of this paper, we leave it for the future.

The risk-free return fits pretty well, falling nearly on the 45-degree line. The estimated expected return rate is 0.3106% per quarter, close to the average realized risk-free return rate, 0.32%. The high fitting accuracy is partly attributed to our estimation design in which the WK distance assigns tremendous weight to the pricing error of the risk-free return. Our model predicts low interest rates because many individuals save. In contrast, in traditional models, every individual wants to borrow, and the interest rates must be high enough to clear the market.

The most significant finding concerns the fitted stock market return. It is near the 45-degree line. The estimated expected stock market return is 2.0625%, close to the average realized stock market return rate of 2.2283% per quarter. The fitted market risk premium computed using the formula (48) is 1.7518% per quarter, which is very close to the realized market risk premium of 1.9083%. Therefore, the proportion of the risk premium that the model accounts for is 91.80%.

Moreover, the estimated volatility of the stochastic discount factor is high. From Equation (38), it is easy to derive Hansen and Jagannathan's (1991) inequality for each test portfolio:

$$\sqrt{var\left[\bar{m}_{t+1}(\hat{\Theta})\right]} \geq E\left[\bar{m}_{t+1}(\hat{\Theta})\right] \frac{E(R_{k,t+1} - R_{0,t+1})}{\sqrt{var(R_{k,t+1} - R_{0,t+1})}}, \quad k = 1, \ldots, K.$$

These inequalities associate the lower bound of the standard deviation of the SDF with the maximum Sharpe ratio of all test portfolios. In our computation, the maximum Sharpe ratio is 0.3515, and the estimated mean and standard deviations of $\bar{m}_{t+1}(\hat{\Theta})$ are 0.9978 and 0.7903, respectively. Thus, the volatility of $\bar{m}_{t+1}(\hat{\Theta})$ is sufficiently large to make all inequalities hold.

5.3.3 Estimated preference

Table 5 presents the estimated preference parameters. We classify the parameters into four groups: (1) time discount factor, (2) distribution parameters of reference points, (3) Bernstein coefficients of the gain function μ_g, and (4) Bernstein coefficients of the loss function μ_g. The estimated subjective time discount factor is 0.9954, which is quite reasonable for quarterly data. The estimated standard deviation (σ_r) of the reference points conditional on observed consumption growth is 0.3044. The number is larger than the standard deviation of individual consumption growth (0.2874, in logarithm). The conditional correlation of the reference points between two adjacent periods ρ is close to zero (-0.0056), suggesting that individuals' reference points are not persistent. The estimates of σ_r and ρ indicate that reference points may be immensely different across individuals and tremendously volatile over time. The estimated values of r_1^1, r_2^1, r_1^2, and r_2^2 are generally not equal to zero, showing some dependence between the unobserved reference points and the observed consumption decisions.

Figure 3 shows the graph of the gain-loss utility function, which comprises two Bernstein polynomials with the coefficients listed in Table 5. The concavity for the gain function and the convexity for the loss function are both clearly visible. Further, it is easy to see that the reference point is a concave kink, implying that individuals are loss averse around the reference point.

To quantitatively characterize the estimated preference, we define the degree of relative risk aversion. Since the gain-loss utility function is flexibly specified, the degree of relative risk aversion is meaningful only locally, varying across the consumption-reference ratio. Let c and θ denote the consumption and reference levels, respectively. As the utility function is $U(c,\theta) = \mu\left(\ln(\frac{c}{\theta})\right)$ and the reference point θ is exogenous, the formula of the degree of relative risk aversion is

$$\gamma\left(\ln(\frac{c}{\theta})\right) \equiv -\frac{U_c''(c,\theta)c}{U_c'(c,\theta)} = 1 - \frac{\mu''(\ln(\frac{c}{\theta}))}{\mu'(\ln(\frac{c}{\theta}))},$$

where the constant one is associated with the log transformation of the consumption-reference ratio. If $\mu^{''}(\ln(\frac{c}{\theta})) < \mu^{'}(\ln(\frac{c}{\theta}))$, the individual is locally risk averse; if $\mu^{''}(\ln(\frac{c}{\theta})) > \mu^{'}(\ln(\frac{c}{\theta}))$, the degree of relative risk aversion is negative (locally risk seeking). We numerically compute the degree of relative risk aversion for each consumption-reference ratio.

Table 5: ESTIMATED PREFERENCE PARAMETERS

Distribution Parameters of Reference Points						Time Discount Factor
σ_r^2	r_1^1	r_2^1	r_1^2	r_2^2	ρ	δ
0.0927	-0.1755	-0.0465	-0.1017	0.1975	-0.0056	0.9954

Bernstein Coefficients of the Gain Function $\mu_g(z)$								
β_0^g	β_1^g	β_2^g	β_3^g	β_4^g	β_5^g	β_6^g	β_7^g	β_8^g
0.0000	1.0000	1.9574	2.8629	3.7065	4.4703	5.1341	5.6709	5.9677

Bernstein Coefficients of the Loss Function $\mu_l(z)$								
β_0^l	β_1^l	β_2^l	β_3^l	β_4^l	β_5^l	β_6^l	β_7^l	β_8^l
-5.3845	-5.2431	-4.9088	-4.4836	-3.9537	-3.3072	-2.5240	-1.4867	0.0000

Notes. This table presents the estimates of preference parameters when the gain-loss utility function μ is restricted to be S-shaped. That is, the gain utility function μ_g is concave, and the loss utility function μ_l is convex. The estimates are obtained by minimizing the WK distance $d(\Theta)$ defined in (45). The distribution parameters of reference points are specified in Equations (30) and (31). Refer to Equations (39) and (40) for the Bernstein polynomials of μ^g and μ^l, respectively. Because of the non-differentiability problem (see Section 5.2), we do not report the standard errors of the estimates.

Figure 4 shows the graph of the degrees of relative risk aversion. Consistent with the prediction of Kahneman and Tversky's (1979) prospect theory, individuals are risk averse toward consumption gains and risk seeking toward consumption losses. If the consumption-reference ratio (in logarithm) is greater than zero, the degree of relative risk aversion is in the interval $[1.2984, 6.6266]$. If this ratio is less than zero, the degree of relative risk aversion belongs to the range $[-8.4680, -0.5590]$. However, it is less likely that an average individual makes a consumption decision near the extreme values of the

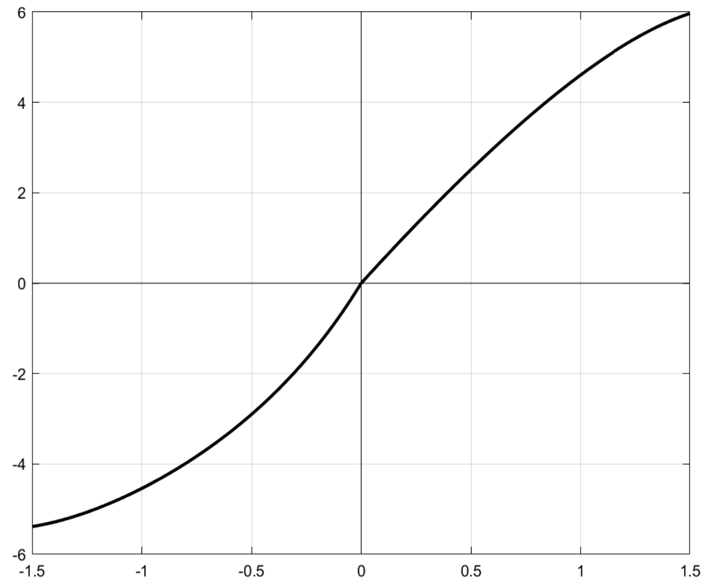

Figure 3: Estimated S-Shaped Gain-Loss Utility Function $\mu(z)$

consumption-reference ratio. A simple computation shows that

$$P\left[\ln\left(\frac{c}{\theta}\right) > 1.2\right] = P\left[\gamma\left(\ln\left(\frac{c}{\theta}\right)\right) > 2.8188\right] = 0.31\%$$

and

$$P\left[\ln\left(\frac{c}{\theta}\right) < -1.2\right] = P\left[\gamma\left(\ln\left(\frac{c}{\theta}\right)\right) < -1.4649\right] = 0.21\%.$$

Therefore, $P\left[\gamma\left(\ln\left(\frac{c}{\theta}\right)\right) \in [-1.4649, 2.8188]\right] = 99.48\%$. In other words, most of the time, individuals are moderately risk averse or risk loving. Some researchers may believe that the degree of relative risk aversion is less than 2. Our computation shows that $P[\gamma\left(\ln\left(\frac{c}{\theta}\right)\right) \geq 2] = 1.64\%$. What distinguishes our model from existing models most is that the probability of one individual being locally risk seeking, $P[\gamma\left(\ln\left(\frac{c}{\theta}\right)\right) \leq 0]$, is 48.09%.

Figure 4 shows that the degree of relative risk aversion is roughly monotonically increasing in the consumption-reference ratio. The monotonicity is strict when this ratio (in logarithm) is positive. This pattern indicates that the diminishing rate of sensitivity to gains increases as the consumption-reference ratio rises. Also, the diminishing rate of sensitivity

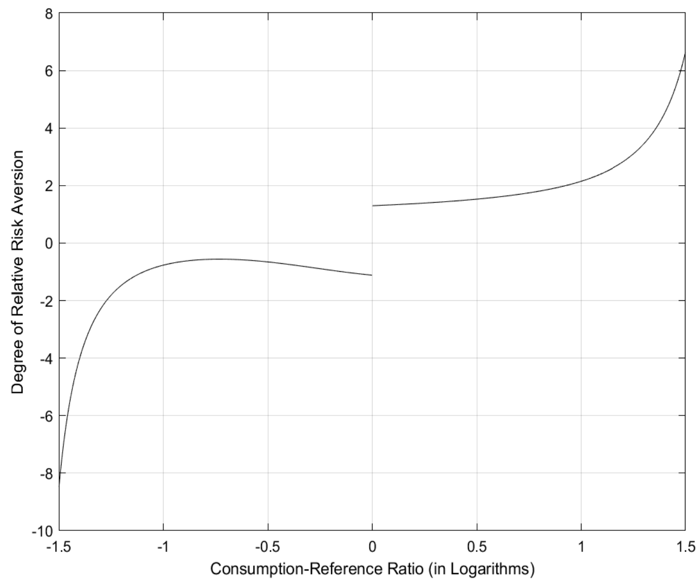

Figure 4: Degree of Relative Risk Aversion

to losses rises as the consumption-reference ratio (near the low extreme) decreases. The pattern of the estimated degree of relative risk aversion helps explain the accelerated expansion. The smaller the sensitivity to gains or losses, the more responsive consumption is to the change in asset returns.

5.4 Concave-shaped utility and the equity premium puzzle

This section examines an alternative asset pricing model based on micro consumption and concave-shaped utility. Though we have rejected concave-shaped consumption utility, it is still meaningful to see how the alternative model generates a high equity premium and how the equity premium puzzle arises in this model. This exercise will further advance our understanding of how an S-shaped consumption utility works.

We specify the alternative model the same as our primary model, except that we restrict the gain-loss utility function to be globally concave shaped. To have a concave specification of $u_l(\cdot)$, we substitute the above condition (V) with $\beta^l_{\nu+2} - 2\beta^l_{\nu+1} + \beta^l_{\nu} \leq 0, \nu = 0, \ldots, n-2$. Also, global concavity requires that the left derivative of the gain-loss utility function μ at the reference point be greater than or equal to its right derivative. Let $\mu'_{-}(0)$ and $\mu'_{+}(0)$ denote the left and right derivatives, respectively. We can compute them by

$$\mu'_{-}(0) \equiv \lim_{z \uparrow 0} u'_{l}(0) = n(\beta^{l}_{n} - \beta^{l}_{n-1}) \frac{1}{(\bar{z} - \underline{z})^{n-1}} (0 - \underline{z})^{n-1},$$

$$\mu'_{+}(0) \equiv \lim_{z \downarrow 0} u'_{g}(0) = n(\beta^{g}_{1} - \beta^{g}_{0}) \frac{1}{(\bar{z} - \underline{z})^{n-1}} (\bar{z})^{n-1}.$$

In the implementation of the estimation, we set $\bar{z} = 1.5$ and $\underline{z} = -1.5$. Therefore, the restriction $\mu'_{-}(0) \geq \mu'_{+}(0)$ holds if and only if $\beta^{g}_{1} \leq -\beta^{l}_{n-1}$.

The alternative model actually fits the data slightly better than our model with an S-shaped consumption utility. First, the estimated WK distance is 0.0969, a little bit smaller than our model's WK distance. Therefore, the alternative model's performance in explaining the cross-section of expected return is also comparable to the CAPM. Second, the fitted risk-free rate is 0.3253%, close to the realized risk-free rate, 0.32%. Third, the fitted equity premium is 2.1372% per quarter, 12% larger than the observed equity premium. Fourth, none of the Hansen-Jagannathan inequalities of the 96 portfolios is rejected.

However, the equity premium puzzle is not solved by the alternative model. The estimated degree of relative risk aversion, given a high consumption-reference ratio, is absurdly large. For example, $\gamma(1.1) = 2.7100$, $\gamma(1.2) = 3.7025$, $\gamma(1.3) = 5.9729$, $\gamma(1.4) = 13.2339$, $\gamma(1.45) = 27.9217$, $\gamma(1.475) = 56.9482$, and $\gamma(1.5) = 2800.0479$. With high levels of risk aversion, it is relatively easy to fit high equity returns or the cross-section of stock returns. See Constantinides (1990), Campbell (1996), Cochrane (1996), and Parker (2003), among others.

We find that the estimated degree of relative risk aversion below the reference point is smaller than that above the reference point. The estimates of γ below the reference point fall within the range $[1.4597, 1.7782]$, while the lowest estimate of γ above the reference point is 1.9947. All of the absurd estimates of γ are associated with high consumption levels. This result differs sharply from Campbell and Cochrane (1999). Their habit model predicts an incredibly high degree of relative risk aversion at low consumption levels. The S-shaped consumption utility can explain this finding. Changing the utility function for losses from our primary convex specification to the alternative concave form may have reduced a large amount of the generated total equity premium. The curvature of the utility function for gains must increase significantly to yield a high equity premium.

6 Conclusion

This paper presents empirical evidence for an S-shaped consumption utility by applying it to solve the equity premium puzzle. Using micro consumption data, we find that the low-quantile consumption growth of many households correlates negatively with equity returns or risk-free returns. This finding rejects a concave-shaped consumption utility because this utility leads to a violation of the law of one price in pricing stocks. Moreover, our empirical evidence supports an S-shaped consumption utility. First, our structural asset pricing model shows that the S-shaped consumption utility can solve the equity premium puzzle. Second, the S-shaped consumption utility can account for the stylized facts in micro consumption, including the "big bang" and "accelerated expansion."

Appendix A Group analysis under an alternative classification Criterion

In this appendix, we investigate the cross-state (i.e., cross-quantile) heterogeneity in s-tockholders' consumption behaviors within several subgroups. We define ten subgroups according to one-period lagged growth rates of stockholders' Type-II consumption. Within each period, the first decile group is labeled Group #1; the second decile group is labeled Group #2; and so forth. Denote $Q_t^g(\tau)$ the τ^{th} quantile of the period t consumption-growth

Table A.1: Quantile regression (consumption of Type II; within all stockholders; grouped by lagged consumption growth)

	#Group=4; $x_t = \log(R^e_{t+1})$			#Group=6; $x_t = \log(R^e_{t+1})$			#Group=8; $x_t = \log(R^e_{t+1})$		
	$\delta^g_1(\tau)$		$sign(\delta^g_1)$	$\delta^g_1(\tau)$		$sign(\delta^g_1)$	$\delta^g_1(\tau)$		$sign(\delta^g_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.2828	0.1333	0.0169	-0.0530	0.1309	0.3429	0.2728	0.1359	0.0223
0.10	-0.2290	0.1206	0.0288	-0.0686	0.1221	0.2870	0.1653	0.1187	0.0818
0.15	-0.2091	0.1002	0.0184	-0.0822	0.1031	0.2125	0.1995	0.1057	0.0296
0.20	-0.2155	0.0928	0.0101	-0.0009	0.0976	0.4965	0.1088	0.0913	0.1166
0.25	-0.1216	0.0879	0.0833	0.0250	0.0937	0.3948	0.0216	0.0914	0.4065
0.30	-0.0682	0.0860	0.2138	0.0138	0.0951	0.4421	0.0631	0.0920	0.2465
0.35	-0.0731	0.0951	0.2211	-0.0042	0.1009	0.4836	0.0356	0.0865	0.3403
0.40	-0.0368	0.0912	0.3432	-0.0452	0.1026	0.3298	0.0006	0.0864	0.4974
0.45	-0.0033	0.0978	0.4864	-0.0170	0.1079	0.4373	0.0496	0.0891	0.2888
0.50	0.0068	0.0985	0.4724	-0.0917	0.1128	0.2082	-0.0122	0.0957	0.4492
0.55	-0.0017	0.1027	0.4935	-0.0729	0.1186	0.2694	-0.0024	0.0968	0.4902
0.60	-0.0113	0.1074	0.4581	-0.0480	0.1110	0.3328	-0.0671	0.1230	0.2928
0.65	-0.0434	0.1177	0.3560	-0.0787	0.1396	0.2865	-0.1264	0.1218	0.1497
0.70	-0.0511	0.1209	0.3362	-0.0176	0.1455	0.4519	-0.1577	0.1426	0.1344
0.75	0.0016	0.1273	0.4951	-0.0201	0.1454	0.4449	-0.1254	0.1479	0.1982
0.80	-0.1511	0.1312	0.1246	-0.0227	0.1619	0.4442	-0.1061	0.1550	0.2468
0.85	-0.1344	0.1397	0.1681	-0.0926	0.1631	0.2851	-0.1613	0.1672	0.1673
0.90	-0.0598	0.1628	0.3567	-0.0853	0.1942	0.3302	-0.1926	0.1866	0.1510
0.95	0.0504	0.1833	0.3917	-0.2088	0.2227	0.1743	-0.2843	0.2009	0.0786

distribution of the g^{th} group. Consider the following simple linear mean regression model

$$Q^g_t(\tau) = \delta^g_0(\tau) + x_t^{'}\delta^g_1(\tau) + \nu^g_t, \qquad t = 1, \ldots, T; g = 1, \ldots, G, \tag{A.1}$$

where x_t can be $\log(R^e_{t+1})$ or $\log(R^f_t)$. The slope $\delta^g_1(\tau)$ represents the correlation between asset returns and the τ^{th} quantile of consumption growth of the g^{th} group. Ju and Li (2021) show that the estimates of (A.1) are asymptotically equivalent to those of a standard quantile regression.

Table A.1 presents the estimation results for some subgroups of all stockholders. The left panel shows that many lower quantiles of consumption growth rates of Group #4 are negatively correlated with stock returns. The middle and the right panels show that there are (weak) negative correlations useful to account for the equity premium puzzle within Group #6 and Group #8.

Table A.2: Quantile regression (consumption of Type II; within young stockholders (age$\leq$30); grouped by lagged consumption growth)

	#Group=3; $x_t = \log(R^e_{t+1})$			#Group=3; $x_t = \log(R^f_t)$			#Group=6; $x_t = \log(R^f_t)$			#Group=7; $x_t = \log(R^e_{t+1})$		
	$\delta^g_1(\tau)$		$sign(\delta^g_1)$	$\delta^g_1(\tau)$		$sign(\delta^g_1)$	$\delta^g_1(\tau)$		$sign(\delta^g_1)$	$\delta^g_1(\tau)$		$sign(\delta^g_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.3180	0.1992	0.0552	-4.0091	1.9926	0.0221	-5.8916	1.7551	0.0004	-0.4265	0.2152	0.0238
0.10	-0.3378	0.1995	0.0452	-4.0908	1.9970	0.0203	-5.7671	1.7589	0.0005	-0.4097	0.2173	0.0297
0.15	-0.3461	0.1987	0.0408	-3.6454	1.9967	0.0339	-6.0295	1.7898	0.0004	-0.4191	0.2188	0.0277
0.20	-0.3126	0.1928	0.0524	-3.8668	1.9290	0.0225	-5.5160	1.8240	0.0012	-0.4030	0.2269	0.0379
0.25	-0.2657	0.2047	0.0972	-3.5890	2.0485	0.0399	-3.7431	1.7562	0.0165	-0.4487	0.2222	0.0217
0.30	-0.1882	0.2134	0.1889	-2.2957	2.1439	0.1421	-3.1860	1.7588	0.0350	-0.4587	0.2360	0.0260
0.35	-0.0721	0.2110	0.3664	-1.7260	2.1187	0.2076	-2.7206	1.7114	0.0560	-0.4904	0.2384	0.0198
0.40	-0.2763	0.2051	0.0890	-1.8532	2.0716	0.1855	-1.9642	1.7331	0.1285	-0.4405	0.2460	0.0367
0.45	-0.2396	0.2195	0.1376	-0.1742	2.2190	0.4687	-3.4113	2.2133	0.0616	-0.2470	0.2437	0.1554
0.50	-0.0508	0.2250	0.4107	0.9981	2.2625	0.3295	-3.6607	2.2891	0.0549	-0.2559	0.2389	0.1421
0.55	0.0812	0.2301	0.3620	0.7333	2.3153	0.3757	-1.8983	2.4830	0.2223	-0.2502	0.2383	0.1468
0.60	0.0835	0.2273	0.3567	1.2058	2.2853	0.2989	-1.6418	2.6564	0.2683	-0.2219	0.2542	0.1914
0.65	0.2070	0.2407	0.1949	3.0093	2.4137	0.1062	0.1136	2.6959	0.4832	-0.0873	0.2874	0.3807
0.70	0.1784	0.2500	0.2378	4.3635	2.4904	0.0399	2.1006	2.7472	0.2222	-0.1862	0.2911	0.2612
0.75	0.1466	0.2463	0.2759	4.4653	2.4495	0.0342	3.6071	2.8659	0.1041	-0.1200	0.2912	0.3402
0.80	0.1945	0.2546	0.2224	5.2045	2.5249	0.0196	4.6847	2.7701	0.0454	-0.0151	0.2894	0.4792
0.85	0.1400	0.2610	0.2958	5.6843	2.5796	0.0138	4.7340	2.7376	0.0419	-0.2166	0.2877	0.2258
0.90	0.2644	0.2690	0.1629	7.5430	2.6331	0.0021	4.9333	2.7663	0.0373	-0.2418	0.2900	0.2023
0.95	0.2769	0.2706	0.1531	7.8052	2.6446	0.0016	4.9912	2.7648	0.0355	-0.2395	0.2894	0.2039

Table A.2 displays the results for some subgroups of all young stockholders. The results for Group #3 and Group #6 indicate that the low quantiles of consumption growth rates of individuals in these groups are negatively correlated with risk-free returns. Also, in Group #3 and Group #7, stock returns vary negatively with the lower quantiles of consumption growth. Most of these negative correlations are statistically significant. Therefore, we conclude that many young stockholders become locally risk-seeking when their consumption level is relatively low.

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Online Appendices to "Identifying S-Shaped Consumption Utility and Solving the Equity Premium Puzzle"

Appendix B Robustness Against Measurement Errors in Consumption

B.1 The errors-in-variables model

Let $c_{i,t}$ and $c^*_{i,t}$ respectively denote the observed and (unobserved) true measures of consumption of the i^{th} individual in period t. Suppose that

$$\log(c_{i,t}) = \log(c^*_{i,t}) + \zeta_{i,t},$$

where $\zeta_{i,t}$ is the measurement error of consumption in logarithm. Taking differencing, we obtain

$$\log(\frac{c_{i,t+1}}{c_{i,t}}) = \log(\frac{c^*_{i,t+1}}{c^*_{i,t}}) + (\zeta_{i,t+1} - \zeta_{i,t}).$$

A similar specification is used by Daniel and Marshall (1997). We assume that measurement errors $(\zeta_{i,t}, \zeta_{i,t+1})$ and true consumption measures $(c^*_{i,t}, c^*_{i,t+1})$ are independent conditional on a vector of macroeconomic variables, which we denote by x_t.

For notational simplicity, let $\eta_{i,t}$ denote $\zeta_{i,t+1} - \zeta_{i,t}$, and $y_{i,t}$ and $y^*_{i,t}$ represent $\log(\frac{c_{i,t+1}}{c_{i,t}})$ and $\log(\frac{c^*_{i,t+1}}{c^*_{i,t}})$, respectively. Thus, we have

$$y_{i,t} = y^*_{i,t} + \eta_{i,t}. \tag{B.1}$$

Our interest is the conditional quantile function of $y^*_{i,t}$ given x_t. However, without further restrictions or extra information, we are unable to obtain the true consumption measure. In the extant literature, approaches that can identify models with classical measurement errors need repeated measurements (Hausman et al. 1991, Li 2002, Schennach 2004a and 2004b), validation data (Hu and Ridder 2004), or instruments (Hausman et al. 1991, Newey 2001, Schennach 2007), none of which are available to us.

Our strategy is to investigate a continuum collection of potentially true measures and check if all of these measures exhibit the same pattern. If all exhibit the big bang and accelerated expansion, for example, and if this collection contains the truth, we are confident that the true consumption exhibits the same phenomena.

We assume that the measurement error is normally distributed with zero mean and a standard deviation of σ_η, i.e.,

$$\eta_{i,t} \sim N(0, \sigma_\eta^2).$$

Suppose $\sigma_\eta \in [0, \sigma_\varepsilon]$ where we allow for σ_ε to be as large as 90% of the standard deviation of y_{it}. The potential measures of consumption growth rates are characterized by the parameters σ_η. We denote the conditional τ-th quantile consumption growth given x_t and σ_η as $Q_{y^*_{i,t}|x_t}(\tau|\sigma_\eta)$. Thus, the collection of distributions of consumption growth under consideration is

$$\left\{ Q_{y^*_{i,t}|x_t}(\tau|\sigma_\eta) : \sigma_\eta \in [0, \sigma_\varepsilon] \right\}. \tag{B.2}$$

In practice, we choose $\sigma_\eta = \rho\sigma_\varepsilon$, $\rho = 0.9, 0.8 \ldots, 0.2, 0.1$. We interpret ρ as the degree of noise severity.

B.2 Conditional deconvolution

We estimate the conditional quantile function of true consumption growth rates using the method of conditional deconvolution.

By the assumption that $\eta_{i,t}$ and y^*_{it} in Equation (B.1) are conditionally independent, we have the conditional deconvolution formula

$$\phi_{y^*_{i,t}|x_t}(s|\sigma_\eta) = \frac{\phi_{y_{i,t}|x_t}(s)}{\phi_{\eta_{i,t}}(s|\sigma_\eta)}, \tag{B.3}$$

where $\phi's$ indicate the conditional characteristic functions. Let i be a complex number such that $i^2 = -1$. The numerator, $\phi_{y_{i,t}|x_t}(s) = E(e^{isy_{i,t}}|x_t)$, can be computed from data using a nonparametric kernel method. The denominator has a closed-form formula: $\phi_{\eta_{i,t}}(s|\sigma_\eta) = e^{-\frac{1}{2}\sigma_\eta^2 s^2}$. Therefore, $\phi_{y^*_{i,t}|x_t}(s|\sigma_\eta)$ is estimable.

Following Gil-Pelaez's (1951) formula

$$F_{y^*_{i,t}|x_t}(\omega|\sigma_\eta) = \frac{1}{2} - \lim_{\chi\to\infty} \int_{-\chi}^{\chi} \frac{e^{-is\omega}}{2\pi is} \phi_{y^*_{i,t}|x_t}(s|\sigma_\eta) ds,$$

we can obtain the corresponding conditional cumulative distribution function (CDF). Let $\hat{\phi}_{y^*_{i,t}|x_t}(s|\sigma_\eta) = \hat{\phi}_{y_{i,t}|x_t}(s)/\phi_{\eta_{i,t}}(s|\sigma_\eta)$ denote the estimated conditional characteristic function.

A natural estimator of $F_{y^*_{i,t}|x_t}(\omega|\sigma_\eta)$ is

$$\hat{F}_{y^*_{i,t}|x_t}(\omega|\sigma_\eta) = \frac{1}{2} + \lim_{\chi\to\infty}\int_{-\chi}^{\chi} \frac{sin(s\omega)RE_{\hat{\phi}_{y_{i,t}|x_t}(s)} - cos(s\omega)IM_{\hat{\phi}_{y_{i,t}|x_t}(s)}}{2\pi\phi_{\eta_{i,t}|x_t}(s)} ds,$$

where RE and IM represent real and imaginary parts of a complex number, respectively.

Further, inverting the estimated conditional CDF leads to an estimate of the conditional quantile function, which we denote by $\hat{Q}_{y^*_{i,t}|x_t}(\tau|\sigma_\eta)$.

B.3 Bandwidth selection

To apply the kernel estimation of the numerator in (B.3), we need to choose optimal bandwidths. Consider the pooled cross-sectional data $\{(y_{i,t}, x_t), t = 1,\dots,T; i = 1,\dots,N\}$ where $y_{i,t}$ is the consumption growth rate, and x_t is a $dimX$-dimensional vector of macroeconomics variables. The kernel estimator of the characteristic function $\phi_{y_{it}}(s|x_t) = E[e^{isy_{it}}|x_t = x]$ is estimated by

$$\hat{\phi}_{y_{it}}(s|x_t = x) = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} e^{isy_{it}} K(x_t, x)}{\sum_{i=1}^{N}\sum_{t=1}^{T} K(x_t, x)},$$

where $K(x_t, x) = \prod_{l=1}^{dimX} k(\frac{x_{tl}-x_l}{h_l})$ and $h_l, l = 1,\dots,dimX$ are bandwidths. Following Ju et al. (2019), we choose bandwidths to minimize

$$CV(h_1,\dots,h_{dimX}) = \int_{-\infty}^{\infty}\left[\sum_{i=1}^{N}\sum_{t=1}^{T}[cos(sy_{it}) - \lambda^{cos}(x_t)]^2 + \sum_{i=1}^{N}\sum_{t=1}^{T}[sin(sy_{it}) - \lambda^{sin}(x_t)]^2\right] ds,$$

where $\lambda^{cos}(x_t)$ and $\lambda^{sin}(x_t)$ are leave-one-out estimates of $E[cos(sy_{it})|x_t]$ and $E[sin(sy_{it})|x_t]$, respectively.

In our empirical application, $T = 121$ and N (for the whole sample) is around 2,600. The sample size is too large to finish running the bandwidth selection algorithm in a reasonable time. To tackle this issue, we first select optimal bandwidths for a smaller subsample and then make an inference of the bandwidths for the original large sample. We write the optimal bandwidth as $h_s^{opt} = c_s \cdot std(x_{t,s}) \cdot A(N,T), s = 1,\dots,dimX$, where $A(N,T)$ is the known order shown later. We first obtain the optimal coefficients c_s for a smaller subsample with, say, $T = 121$ and $N = 200$. The observations in each period of the subsample are randomly drawn from the corresponding period of the large sample. We repeat this procedure 100 times and calculate the median (denoted by c_s^*) of these coefficients. Then, we take $h_s^{opt} = c_s^* \cdot std(x_{t,s}) \cdot A(2600,121), s = 1,\dots,dimX$ as the optimal bandwidths for the large sample.

Now, let us derive the order of the optimal bandwidths. Consider the model

$$y_{it} = E(y_{it}|x_t) + u_{it} = g(x_t) + u_{it}; t = 1, \ldots, T; i = 1, \ldots, N.$$

The nonparametric kernel estimator is

$$\hat{g}(x) = \frac{\frac{1}{NTh_1 \ldots h_{dimX}} \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it} K(\frac{x_t - x}{h})}{\hat{f}(x)},$$

where the denominator $\hat{f}(x) = \frac{1}{Th_1,\ldots,h_{dimX}} \sum_{t=1}^{T} K(\frac{x_t - x}{h})$ is the density estimator. We write $\hat{g}(x) - g(x) = \frac{[\hat{g}(x) - g(x)]\hat{f}(x)}{\hat{f}(x)} \equiv \frac{\hat{m}(x)}{\hat{f}(x)}$, where $\hat{m}(x) = [\hat{g}(x) - g(x)]\hat{f}(x) \equiv \hat{m}_1(x) + \hat{m}_2(x)$, and

$$\hat{m}_1(x) = \frac{1}{NTh_1 \ldots h_{dimX}} \sum_{i=1}^{N} \sum_{t=1}^{T} [g(x_t) - g(x)] K(\frac{x_t - x}{h}) = \frac{1}{Th_1, \ldots, h_{dimX}} \sum_{t=1}^{T} [g(x_t) - g(x)] K(\frac{x_t - x}{h}),$$

$$\hat{m}_2(x) = \frac{1}{NTh_1 \ldots h_{dimX}} \sum_{i=1}^{N} \sum_{t=1}^{T} u_{it} K(\frac{x_t - x}{h})$$

Following the derivation on page 62 of the textbook by Li and Racine (2007), it is easy to get

$$\hat{m}_1(x) = f(x) \sum_{s=1}^{dimX} B_s(x) h_s^2 + O_p(\eta_2^{3/2} + \eta_2^{1/2} \eta_1^{1/2}),$$

where $\eta_1 = (Th_1 \ldots h_{dimX})^{-1}$ and $\eta_2 = \sum_{s=1}^{dimX} h_s^2$. Furthermore, it is easy to obtain $E[\hat{m}_2(x)]^2 = \eta_3 \Omega(x) + s.o.$, where $\eta_3 = (NTh_1 \ldots h_{dimX})^{-1}$. Hence, we have

$$\hat{m}(x) = \hat{m}_1(x) + \hat{m}_2(x) = f(x) \sum_{s=1}^{dimX} B_s(x) h_s^2 + O_p(\eta_2^{3/2} + \eta_2^{1/2} \eta_1^{1/2} + \eta_3^{1/2}).$$

As x_t in our application is of one dimension, then

$$E[\hat{g}(x) - g(x)]^2 = O(h^4 + \frac{h}{T} + \frac{1}{NTh}).$$

If $\frac{1}{T} = o(h^3)$, then $h \sim (NT)^{-1/5}$. This implies that $\frac{1}{T} = o((NT)^{-3/5})$, which is false since $N >> T$. Thus, h^4 is a small term with an order larger than that of h/T. Therefore, $h_1^{opt} = c_1 \cdot std(x_t) N^{-1/2}$.

B.4 Estimation and results

Let $\hat{Q}_{y^*_{i,t}|x_t}(\tau|\sigma_\eta)$ denote the estimator of the conditional quantile function $Q_{y^*_{i,t}|x_t}(\tau|\sigma_\eta)$. In the spirit of the equivalence between Model (19) and Model (20), we specify a linear model as

$$\hat{Q}_{y^*_{i,t}|x_t}(\tau|\sigma_\eta) = \delta_0(\tau|\sigma_\eta) + x_t'\delta_1(\tau|\sigma_\eta) + \nu_t, \qquad t = 1, \ldots, T, \tag{B.4}$$

where $\sigma_\eta \in [0, \sigma_\varepsilon]$ and ν_t denotes the approximation errors. The slope coefficient $\delta_1(\tau|\sigma_\eta)$ reflects the correlation between x_t and the τ^{th} quantile of consumption growth rates. Every time we run the linear regression, we fix the values of σ_η and τ. Given their values, the dependent variable is a function of x_t. Thus, the linear model (B.4) is identified using time variations. The simple ordinary least squares estimation yields good estimates of $\delta_0(\tau|\sigma_\eta)$ and $\delta_1(\tau|\sigma_\eta)$. For statistical inference, we apply bootstrapping using sampling with replacement (replicated 250 times).

Corresponding to the specifications of the standard quantile regression (19), we specify $y^*_{i,t}$ in (B.4) as the logarithm of consumption growth rates of Type I, II, or III. The regressor x_t is $\log(R^e_{t+1})$ or $\log(R^f_t)$. Tables B.1 – B.6 present the estimation results for the whole sample. Also, Tables B.7 and B.8 present the estimation results of regression (B.4) for Type-II consumption of young households.

As expected, we find that measurement errors affect the magnitudes of the parameter estimation. For example, in Table B.4, the estimate of the slope coefficient $\delta_1(\tau|\sigma_\eta)$ at quantile $\tau = 0.05$ and $\sigma_\eta = 0.9\sigma_\varepsilon$ (i.e., the standard deviation of the measurement errors is as large as 81% of that of the dependent variable) is -3.3677. As the noise gets smaller, its magnitude becomes larger. When $\sigma_\eta = 0.1\sigma_\varepsilon$, the estimate has a value of -9.1119.

However, our estimates suggest that the patterns of estimated $\delta_1(\tau|\sigma_\eta)$ revealed in the standard regression assuming without measurement error remain unchanged. In particular, the big bang and accelerated expansion are still prominent in all specifications of measurement errors whenever these phenomena are observed from the corresponding standard quantile estimation. Therefore, our results are robust against classic measurement errors. This robustness is conceivable as Ju and Li (2021) showed by simulations that the big bang does not disappear even though the measurement errors may result in some biases in estimating the quantile regression coefficients.

Table B.1: Quantile Regression (Whole Sample; Type-I Consumption; $x_t = \log(R^e_{t+1})$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0250	0.0116	0.0080	-0.0263	0.0129	0.0120	-0.0263	0.0147	0.0200
0.10	-0.0200	0.0103	0.0200	-0.0209	0.0108	0.0240	-0.0211	0.0114	0.0240
0.15	-0.0159	0.0095	0.0480	-0.0163	0.0098	0.0440	-0.0163	0.0101	0.0440
0.20	-0.0123	0.0091	0.0920	-0.0124	0.0092	0.0920	-0.0122	0.0094	0.1080
0.25	-0.0090	0.0087	0.1440	-0.0089	0.0088	0.1600	-0.0086	0.0089	0.1720
0.30	-0.0060	0.0085	0.2160	-0.0057	0.0085	0.2280	-0.0053	0.0086	0.2400
0.35	-0.0032	0.0083	0.3360	-0.0027	0.0083	0.3600	-0.0022	0.0084	0.3880
0.40	-0.0004	0.0082	0.4920	0.0001	0.0082	0.4720	0.0007	0.0082	0.4440
0.45	0.0023	0.0082	0.3760	0.0030	0.0081	0.3520	0.0036	0.0081	0.3400
0.50	0.0050	0.0081	0.2760	0.0058	0.0081	0.2440	0.0066	0.0081	0.1960
0.55	0.0078	0.0081	0.1600	0.0088	0.0081	0.1440	0.0096	0.0081	0.1280
0.60	0.0107	0.0082	0.0880	0.0118	0.0082	0.0640	0.0128	0.0082	0.0440
0.65	0.0137	0.0083	0.0400	0.0151	0.0083	0.0280	0.0162	0.0083	0.0240
0.70	0.0170	0.0085	0.0240	0.0187	0.0085	0.0160	0.0200	0.0085	0.0120
0.75	0.0206	0.0087	0.0120	0.0227	0.0088	0.0120	0.0244	0.0088	0.0120
0.80	0.0248	0.0090	0.0120	0.0275	0.0092	0.0040	0.0297	0.0093	0.0040
0.85	0.0298	0.0095	0.0040	0.0334	0.0098	0.0040	0.0364	0.0100	0.0000
0.90	0.0360	0.0103	0.0000	0.0413	0.0108	0.0000	0.0460	0.0113	0.0000
0.95	0.0445	0.0115	0.0000	0.0535	0.0127	0.0000	0.0629	0.0143	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0249	0.0169	0.0680	-0.0218	0.0222	0.1360	-0.0196	0.0244	0.1920
0.10	-0.0210	0.0119	0.0200	-0.0206	0.0123	0.0280	-0.0200	0.0126	0.0440
0.15	-0.0160	0.0103	0.0480	-0.0156	0.0105	0.0720	-0.0151	0.0106	0.0800
0.20	-0.0118	0.0095	0.1240	-0.0113	0.0096	0.1280	-0.0109	0.0097	0.1360
0.25	-0.0082	0.0090	0.1800	-0.0077	0.0090	0.2000	-0.0073	0.0091	0.2000
0.30	-0.0048	0.0086	0.2560	-0.0044	0.0087	0.2760	-0.0039	0.0087	0.3040
0.35	-0.0017	0.0084	0.4280	-0.0013	0.0084	0.4360	-0.0008	0.0084	0.4480
0.40	0.0013	0.0082	0.4280	0.0017	0.0083	0.4200	0.0021	0.0083	0.3960
0.45	0.0042	0.0081	0.2960	0.0047	0.0082	0.2720	0.0051	0.0082	0.2520
0.50	0.0072	0.0081	0.1760	0.0077	0.0081	0.1720	0.0081	0.0081	0.1520
0.55	0.0103	0.0081	0.0960	0.0108	0.0081	0.0680	0.0113	0.0082	0.0600
0.60	0.0135	0.0082	0.0360	0.0141	0.0082	0.0360	0.0146	0.0082	0.0360
0.65	0.0171	0.0083	0.0160	0.0178	0.0084	0.0160	0.0183	0.0084	0.0160
0.70	0.0211	0.0086	0.0120	0.0218	0.0086	0.0120	0.0224	0.0086	0.0120
0.75	0.0257	0.0089	0.0080	0.0266	0.0089	0.0080	0.0273	0.0090	0.0080
0.80	0.0313	0.0094	0.0040	0.0326	0.0095	0.0040	0.0334	0.0095	0.0000
0.85	0.0388	0.0102	0.0000	0.0406	0.0104	0.0000	0.0418	0.0105	0.0000
0.90	0.0499	0.0118	0.0000	0.0530	0.0122	0.0000	0.0553	0.0125	0.0000
0.95	0.0723	0.0161	0.0000	0.0804	0.0192	0.0040	0.0870	0.0210	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0192	0.0251	0.2160	-0.0195	0.0253	0.2360	-0.0198	0.0252	0.2240
0.10	-0.0195	0.0128	0.0520	-0.0191	0.0130	0.0560	-0.0189	0.0131	0.0640
0.15	-0.0146	0.0107	0.0960	-0.0143	0.0108	0.1000	-0.0141	0.0108	0.1000
0.20	-0.0105	0.0097	0.1480	-0.0102	0.0098	0.1600	-0.0100	0.0098	0.1640
0.25	-0.0069	0.0091	0.2040	-0.0066	0.0091	0.2080	-0.0064	0.0091	0.2120
0.30	-0.0036	0.0087	0.3280	-0.0033	0.0087	0.3400	-0.0032	0.0088	0.3480
0.35	-0.0005	0.0085	0.4680	-0.0003	0.0085	0.4800	-0.0001	0.0085	0.4920
0.40	0.0025	0.0083	0.3840	0.0027	0.0083	0.3720	0.0028	0.0083	0.3680
0.45	0.0054	0.0082	0.2400	0.0057	0.0082	0.2360	0.0058	0.0082	0.2360
0.50	0.0084	0.0082	0.1480	0.0087	0.0082	0.1480	0.0088	0.0082	0.1440
0.55	0.0116	0.0082	0.0520	0.0118	0.0082	0.0520	0.0119	0.0082	0.0560
0.60	0.0149	0.0082	0.0280	0.0152	0.0083	0.0240	0.0153	0.0083	0.0200
0.65	0.0186	0.0084	0.0120	0.0189	0.0084	0.0120	0.0190	0.0084	0.0120
0.70	0.0228	0.0086	0.0120	0.0231	0.0086	0.0120	0.0232	0.0087	0.0120
0.75	0.0278	0.0090	0.0040	0.0281	0.0090	0.0040	0.0283	0.0090	0.0040
0.80	0.0341	0.0096	0.0000	0.0344	0.0096	0.0000	0.0347	0.0096	0.0000
0.85	0.0427	0.0106	0.0000	0.0433	0.0107	0.0000	0.0436	0.0107	0.0000
0.90	0.0569	0.0127	0.0000	0.0578	0.0128	0.0000	0.0584	0.0129	0.0000
0.95	0.0922	0.0220	0.0000	0.0958	0.0227	0.0000	0.0978	0.0230	0.0000

Table B.2: Quantile Regression (Whole Sample; Type-I Consumption; $x_t = \log(R_t^f)$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.7390	0.1096	0.0000	-1.0642	0.1228	0.0000	-1.3912	0.1392	0.0000
0.10	-0.3493	0.0972	0.0000	-0.5295	0.1029	0.0000	-0.6799	0.1089	0.0000
0.15	-0.0609	0.0899	0.2240	-0.1754	0.0928	0.0480	-0.2629	0.0958	0.0000
0.20	0.1724	0.0850	0.0160	0.0949	0.0866	0.1400	0.0390	0.0881	0.3200
0.25	0.3717	0.0818	0.0000	0.3182	0.0824	0.0000	0.2809	0.0833	0.0000
0.30	0.5493	0.0793	0.0000	0.5125	0.0796	0.0000	0.4877	0.0800	0.0000
0.35	0.7121	0.0777	0.0000	0.6882	0.0776	0.0000	0.6725	0.0778	0.0000
0.40	0.8656	0.0766	0.0000	0.8520	0.0764	0.0000	0.8434	0.0763	0.0000
0.45	1.0133	0.0760	0.0000	1.0087	0.0756	0.0000	1.0062	0.0755	0.0000
0.50	1.1583	0.0758	0.0000	1.1623	0.0753	0.0000	1.1654	0.0752	0.0000
0.55	1.3034	0.0760	0.0000	1.3160	0.0755	0.0000	1.3247	0.0754	0.0000
0.60	1.4516	0.0766	0.0000	1.4733	0.0762	0.0000	1.4882	0.0761	0.0000
0.65	1.6059	0.0776	0.0000	1.6380	0.0774	0.0000	1.6603	0.0774	0.0000
0.70	1.7699	0.0792	0.0000	1.8151	0.0792	0.0000	1.8465	0.0795	0.0000
0.75	1.9490	0.0815	0.0000	2.0113	0.0819	0.0000	2.0554	0.0825	0.0000
0.80	2.1504	0.0847	0.0000	2.2369	0.0859	0.0000	2.3002	0.0870	0.0000
0.85	2.3860	0.0893	0.0000	2.5102	0.0916	0.0000	2.6055	0.0939	0.0000
0.90	2.6771	0.0962	0.0000	2.8674	0.1011	0.0000	3.0258	0.1060	0.0000
0.95	3.0686	0.1079	0.0000	3.4015	0.1194	0.0000	3.7308	0.1334	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-1.7029	0.1582	0.0000	-1.9788	0.1806	0.0000	-2.1911	0.1988	0.0000
0.10	-0.7960	0.1144	0.0000	-0.8780	0.1190	0.0000	-0.9308	0.1226	0.0000
0.15	-0.3248	0.0982	0.0000	-0.3652	0.1005	0.0000	-0.3893	0.1021	0.0000
0.20	0.0014	0.0895	0.4680	-0.0218	0.0907	0.4040	-0.0348	0.0916	0.3400
0.25	0.2569	0.0840	0.0000	0.2427	0.0847	0.0080	0.2351	0.0853	0.0120
0.30	0.4722	0.0804	0.0000	0.4634	0.0808	0.0000	0.4590	0.0812	0.0000
0.35	0.6629	0.0780	0.0000	0.6578	0.0782	0.0000	0.6555	0.0785	0.0000
0.40	0.8384	0.0764	0.0000	0.8361	0.0766	0.0000	0.8352	0.0768	0.0000
0.45	1.0051	0.0755	0.0000	1.0049	0.0756	0.0000	1.0053	0.0757	0.0000
0.50	1.1678	0.0752	0.0000	1.1697	0.0752	0.0000	1.1710	0.0754	0.0000
0.55	1.3307	0.0754	0.0000	1.3346	0.0755	0.0000	1.3370	0.0756	0.0000
0.60	1.4980	0.0761	0.0000	1.5042	0.0762	0.0000	1.5078	0.0764	0.0000
0.65	1.6747	0.0775	0.0000	1.6836	0.0777	0.0000	1.6887	0.0779	0.0000
0.70	1.8672	0.0798	0.0000	1.8798	0.0800	0.0000	1.8870	0.0803	0.0000
0.75	2.0848	0.0830	0.0000	2.1030	0.0835	0.0000	2.1134	0.0839	0.0000
0.80	2.3433	0.0880	0.0000	2.3707	0.0888	0.0000	2.3867	0.0895	0.0000
0.85	2.6733	0.0960	0.0000	2.7180	0.0977	0.0000	2.7453	0.0988	0.0000
0.90	3.1478	0.1105	0.0000	3.2343	0.1142	0.0000	3.2905	0.1170	0.0000
0.95	4.0397	0.1497	0.0000	4.3095	0.1684	0.0000	4.5194	0.1837	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-2.3282	0.2081	0.0000	-2.4061	0.2142	0.0000	-2.4433	0.2174	0.0000
0.10	-0.9619	0.1252	0.0000	-0.9785	0.1268	0.0000	-0.9861	0.1278	0.0000
0.15	-0.4024	0.1033	0.0000	-0.4088	0.1040	0.0000	-0.4115	0.1045	0.0000
0.20	-0.0415	0.0923	0.3240	-0.0445	0.0927	0.3040	-0.0454	0.0930	0.3040
0.25	0.2315	0.0857	0.0120	0.2303	0.0860	0.0120	0.2299	0.0861	0.0120
0.30	0.4572	0.0815	0.0000	0.4567	0.0817	0.0000	0.4566	0.0818	0.0000
0.35	0.6547	0.0787	0.0000	0.6547	0.0789	0.0000	0.6549	0.0790	0.0000
0.40	0.8352	0.0769	0.0000	0.8354	0.0770	0.0000	0.8357	0.0771	0.0000
0.45	1.0058	0.0759	0.0000	1.0063	0.0760	0.0000	1.0066	0.0761	0.0000
0.50	1.1720	0.0754	0.0000	1.1726	0.0756	0.0000	1.1730	0.0757	0.0000
0.55	1.3384	0.0757	0.0000	1.3393	0.0757	0.0000	1.3397	0.0758	0.0000
0.60	1.5098	0.0765	0.0000	1.5108	0.0766	0.0000	1.5113	0.0767	0.0000
0.65	1.6915	0.0780	0.0000	1.6928	0.0781	0.0000	1.6934	0.0782	0.0000
0.70	1.8908	0.0805	0.0000	1.8926	0.0806	0.0000	1.8933	0.0807	0.0000
0.75	2.1190	0.0842	0.0000	2.1216	0.0844	0.0000	2.1227	0.0845	0.0000
0.80	2.3954	0.0900	0.0000	2.3996	0.0903	0.0000	2.4014	0.0905	0.0000
0.85	2.7605	0.0998	0.0000	2.7683	0.1003	0.0000	2.7717	0.1007	0.0000
0.90	3.3240	0.1190	0.0000	3.3420	0.1204	0.0000	3.3504	0.1211	0.0000
0.95	4.6621	0.1934	0.0000	4.7463	0.1992	0.0000	4.7885	0.2025	0.0000

Table B.3: Quantile Regression (Whole Sample; Type-III Consumption; $x_t = \log(R^e_{t+1})$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.1068	0.0282	0.0000	-0.1139	0.0482	0.0080	-0.0912	0.1123	0.0920
0.10	-0.0875	0.0222	0.0000	-0.0962	0.0246	0.0000	-0.1026	0.0269	0.0000
0.15	-0.0714	0.0193	0.0000	-0.0775	0.0205	0.0000	-0.0818	0.0217	0.0000
0.20	-0.0578	0.0176	0.0000	-0.0621	0.0183	0.0000	-0.0651	0.0190	0.0000
0.25	-0.0459	0.0164	0.0000	-0.0490	0.0169	0.0000	-0.0511	0.0173	0.0000
0.30	-0.0352	0.0157	0.0040	-0.0375	0.0159	0.0040	-0.0389	0.0162	0.0040
0.35	-0.0253	0.0151	0.0520	-0.0269	0.0153	0.0480	-0.0279	0.0155	0.0440
0.40	-0.0160	0.0148	0.1600	-0.0170	0.0149	0.1520	-0.0176	0.0150	0.1400
0.45	-0.0069	0.0146	0.3080	-0.0074	0.0147	0.2960	-0.0077	0.0148	0.2920
0.50	0.0020	0.0146	0.4480	0.0019	0.0146	0.4600	0.0019	0.0147	0.4480
0.55	0.0109	0.0147	0.2040	0.0113	0.0147	0.2040	0.0116	0.0148	0.2080
0.60	0.0199	0.0149	0.0880	0.0209	0.0150	0.0880	0.0215	0.0150	0.0680
0.65	0.0294	0.0153	0.0280	0.0309	0.0154	0.0280	0.0320	0.0155	0.0240
0.70	0.0394	0.0159	0.0160	0.0417	0.0160	0.0080	0.0433	0.0161	0.0080
0.75	0.0503	0.0166	0.0000	0.0537	0.0169	0.0000	0.0560	0.0171	0.0000
0.80	0.0626	0.0177	0.0000	0.0674	0.0181	0.0000	0.0708	0.0185	0.0000
0.85	0.0769	0.0192	0.0000	0.0840	0.0200	0.0000	0.0894	0.0207	0.0000
0.90	0.0945	0.0216	0.0000	0.1058	0.0231	0.0000	0.1152	0.0246	0.0000
0.95	0.1183	0.0257	0.0000	0.1389	0.0297	0.0000	0.1627	0.0369	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0662	0.1181	0.2320	-0.0866	0.0877	0.1840	-0.1101	0.0747	0.0840
0.10	-0.1073	0.0290	0.0000	-0.1106	0.0305	0.0000	-0.1128	0.0315	0.0000
0.15	-0.0847	0.0227	0.0000	-0.0864	0.0234	0.0000	-0.0874	0.0240	0.0000
0.20	-0.0669	0.0196	0.0000	-0.0679	0.0201	0.0000	-0.0684	0.0204	0.0000
0.25	-0.0523	0.0177	0.0000	-0.0529	0.0181	0.0000	-0.0532	0.0183	0.0000
0.30	-0.0397	0.0165	0.0080	-0.0401	0.0168	0.0080	-0.0403	0.0170	0.0160
0.35	-0.0284	0.0157	0.0400	-0.0287	0.0159	0.0440	-0.0287	0.0161	0.0440
0.40	-0.0179	0.0152	0.1400	-0.0181	0.0154	0.1400	-0.0181	0.0155	0.1400
0.45	-0.0079	0.0149	0.3040	-0.0079	0.0150	0.3040	-0.0080	0.0152	0.3040
0.50	0.0019	0.0148	0.4520	0.0019	0.0149	0.4560	0.0020	0.0150	0.4480
0.55	0.0118	0.0149	0.2200	0.0119	0.0150	0.2280	0.0119	0.0151	0.2240
0.60	0.0219	0.0151	0.0680	0.0221	0.0152	0.0720	0.0222	0.0153	0.0760
0.65	0.0326	0.0156	0.0240	0.0329	0.0157	0.0200	0.0331	0.0158	0.0200
0.70	0.0443	0.0163	0.0080	0.0448	0.0164	0.0080	0.0450	0.0166	0.0080
0.75	0.0574	0.0173	0.0000	0.0583	0.0175	0.0000	0.0587	0.0177	0.0000
0.80	0.0731	0.0189	0.0000	0.0745	0.0192	0.0000	0.0752	0.0194	0.0000
0.85	0.0932	0.0214	0.0000	0.0956	0.0219	0.0000	0.0970	0.0223	0.0000
0.90	0.1226	0.0260	0.0000	0.1278	0.0271	0.0000	0.1311	0.0279	0.0000
0.95	0.1914	0.0457	0.0000	0.2160	0.0498	0.0000	0.2296	0.0516	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.1309	0.0692	0.0240	-0.1456	0.0668	0.0120	-0.1540	0.0658	0.0120
0.10	-0.1142	0.0322	0.0000	-0.1149	0.0326	0.0000	-0.1153	0.0328	0.0000
0.15	-0.0878	0.0244	0.0000	-0.0879	0.0246	0.0000	-0.0880	0.0248	0.0000
0.20	-0.0685	0.0207	0.0000	-0.0685	0.0209	0.0000	-0.0685	0.0210	0.0000
0.25	-0.0532	0.0185	0.0000	-0.0532	0.0187	0.0000	-0.0531	0.0188	0.0000
0.30	-0.0403	0.0171	0.0200	-0.0402	0.0172	0.0200	-0.0402	0.0173	0.0200
0.35	-0.0287	0.0162	0.0480	-0.0287	0.0163	0.0480	-0.0286	0.0164	0.0480
0.40	-0.0181	0.0156	0.1360	-0.0180	0.0157	0.1400	-0.0180	0.0157	0.1400
0.45	-0.0079	0.0152	0.3120	-0.0079	0.0153	0.3080	-0.0079	0.0154	0.3160
0.50	0.0020	0.0151	0.4520	0.0020	0.0152	0.4520	0.0020	0.0152	0.4520
0.55	0.0120	0.0152	0.2240	0.0120	0.0152	0.2280	0.0120	0.0153	0.2280
0.60	0.0222	0.0154	0.0760	0.0222	0.0155	0.0760	0.0222	0.0155	0.0760
0.65	0.0331	0.0159	0.0200	0.0331	0.0160	0.0200	0.0331	0.0160	0.0200
0.70	0.0451	0.0167	0.0080	0.0451	0.0167	0.0080	0.0451	0.0168	0.0080
0.75	0.0589	0.0178	0.0000	0.0589	0.0179	0.0000	0.0589	0.0179	0.0000
0.80	0.0755	0.0196	0.0000	0.0757	0.0197	0.0000	0.0757	0.0198	0.0000
0.85	0.0978	0.0225	0.0000	0.0981	0.0227	0.0000	0.0982	0.0228	0.0000
0.90	0.1329	0.0285	0.0000	0.1338	0.0289	0.0000	0.1342	0.0291	0.0000
0.95	0.2366	0.0531	0.0000	0.2399	0.0541	0.0000	0.2411	0.0547	0.0000

Table B.4: Quantile Regression (Whole Sample; Type-III Consumption; $x_t = \log(R_t^f)$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-3.3677	0.2771	0.0000	-4.5378	0.3274	0.0000	-5.8637	0.5501	0.0000
0.10	-2.0935	0.2389	0.0000	-2.7008	0.2547	0.0000	-3.2079	0.2703	0.0000
0.15	-1.2004	0.2183	0.0000	-1.5868	0.2266	0.0000	-1.8831	0.2344	0.0000
0.20	-0.4982	0.2049	0.0120	-0.7656	0.2099	0.0000	-0.9617	0.2145	0.0000
0.25	0.0916	0.1954	0.3120	-0.1016	0.1986	0.3240	-0.2399	0.2016	0.1080
0.30	0.6097	0.1885	0.0000	0.4676	0.1906	0.0040	0.3669	0.1927	0.0520
0.35	1.0804	0.1834	0.0000	0.9763	0.1848	0.0000	0.9023	0.1863	0.0000
0.40	1.5199	0.1796	0.0000	1.4460	0.1806	0.0000	1.3926	0.1818	0.0000
0.45	1.9398	0.1770	0.0000	1.8919	0.1777	0.0000	1.8555	0.1788	0.0000
0.50	2.3497	0.1753	0.0000	2.3254	0.1759	0.0000	2.3043	0.1770	0.0000
0.55	2.7579	0.1746	0.0000	2.7567	0.1753	0.0000	2.7505	0.1764	0.0000
0.60	3.1725	0.1748	0.0000	3.1957	0.1758	0.0000	3.2054	0.1770	0.0000
0.65	3.6025	0.1762	0.0000	3.6535	0.1775	0.0000	3.6814	0.1792	0.0000
0.70	4.0588	0.1788	0.0000	4.1437	0.1808	0.0000	4.1947	0.1830	0.0000
0.75	4.5560	0.1831	0.0000	4.6856	0.1861	0.0000	4.7681	0.1892	0.0000
0.80	5.1156	0.1896	0.0000	5.3090	0.1945	0.0000	5.4396	0.1991	0.0000
0.85	5.7727	0.1999	0.0000	6.0668	0.2079	0.0000	6.2798	0.2156	0.0000
0.90	6.5927	0.2165	0.0000	7.0699	0.2310	0.0000	7.4527	0.2455	0.0000
0.95	7.7238	0.2476	0.0000	8.6306	0.2823	0.0000	9.5331	0.3251	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-7.1289	0.5945	0.0000	-8.0269	0.5740	0.0000	-8.6054	0.5689	0.0000
0.10	-3.5943	0.2840	0.0000	-3.8604	0.2951	0.0000	-4.0253	0.3031	0.0000
0.15	-2.0926	0.2412	0.0000	-2.2284	0.2463	0.0000	-2.3092	0.2502	0.0000
0.20	-1.0954	0.2185	0.0000	-1.1799	0.2216	0.0000	-1.2293	0.2240	0.0000
0.25	-0.3326	0.2043	0.0360	-0.3908	0.2065	0.0280	-0.4250	0.2081	0.0160
0.30	0.2994	0.1946	0.0760	0.2568	0.1963	0.0920	0.2314	0.1976	0.1080
0.35	0.8522	0.1878	0.0000	0.8198	0.1892	0.0000	0.7997	0.1903	0.0000
0.40	1.3554	0.1831	0.0000	1.3303	0.1843	0.0000	1.3140	0.1853	0.0000
0.45	1.8287	0.1800	0.0000	1.8093	0.1811	0.0000	1.7957	0.1821	0.0000
0.50	2.2866	0.1782	0.0000	2.2722	0.1793	0.0000	2.2608	0.1803	0.0000
0.55	2.7416	0.1777	0.0000	2.7318	0.1788	0.0000	2.7224	0.1799	0.0000
0.60	3.2058	0.1785	0.0000	3.2009	0.1798	0.0000	3.1936	0.1809	0.0000
0.65	3.6928	0.1809	0.0000	3.6937	0.1824	0.0000	3.6889	0.1838	0.0000
0.70	4.2203	0.1851	0.0000	4.2289	0.1871	0.0000	4.2278	0.1886	0.0000
0.75	4.8141	0.1921	0.0000	4.8344	0.1947	0.0000	4.8391	0.1967	0.0000
0.80	5.5181	0.2035	0.0000	5.5581	0.2070	0.0000	5.5731	0.2098	0.0000
0.85	6.4183	0.2223	0.0000	6.4972	0.2279	0.0000	6.5343	0.2322	0.0000
0.90	7.7300	0.2586	0.0000	7.9084	0.2693	0.0000	8.0073	0.2775	0.0000
0.95	10.3776	0.3787	0.0000	11.0959	0.4504	0.0000	11.5776	0.4725	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\mid\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-8.9201	0.5681	0.0000	-9.0616	0.5680	0.0000	-9.1119	0.5681	0.0000
0.10	-4.1168	0.3086	0.0000	-4.1614	0.3120	0.0000	-4.1800	0.3138	0.0000
0.15	-2.3527	0.2529	0.0000	-2.3736	0.2546	0.0000	-2.3823	0.2556	0.0000
0.20	-1.2558	0.2257	0.0000	-1.2686	0.2269	0.0000	-1.2740	0.2275	0.0000
0.25	-0.4435	0.2094	0.0160	-0.4528	0.2103	0.0160	-0.4568	0.2107	0.0160
0.30	0.2170	0.1986	0.1280	0.2095	0.1993	0.1440	0.2061	0.1997	0.1480
0.35	0.7879	0.1912	0.0000	0.7813	0.1919	0.0000	0.7781	0.1923	0.0000
0.40	1.3037	0.1862	0.0000	1.2975	0.1868	0.0000	1.2943	0.1871	0.0000
0.45	1.7864	0.1829	0.0000	1.7804	0.1835	0.0000	1.7770	0.1838	0.0000
0.50	2.2521	0.1811	0.0000	2.2461	0.1817	0.0000	2.2426	0.1821	0.0000
0.55	2.7143	0.1808	0.0000	2.7082	0.1814	0.0000	2.7045	0.1817	0.0000
0.60	3.1861	0.1819	0.0000	3.1798	0.1826	0.0000	3.1758	0.1830	0.0000
0.65	3.6822	0.1848	0.0000	3.6759	0.1855	0.0000	3.6715	0.1860	0.0000
0.70	4.2222	0.1899	0.0000	4.2159	0.1908	0.0000	4.2114	0.1913	0.0000
0.75	4.8357	0.1982	0.0000	4.8298	0.1994	0.0000	4.8249	0.2000	0.0000
0.80	5.5742	0.2118	0.0000	5.5696	0.2133	0.0000	5.5650	0.2140	0.0000
0.85	6.5460	0.2353	0.0000	6.5457	0.2373	0.0000	6.5421	0.2386	0.0000
0.90	8.0523	0.2835	0.0000	8.0667	0.2872	0.0000	8.0684	0.2893	0.0000
0.95	11.8426	0.4834	0.0000	11.9563	0.4898	0.0000	11.9922	0.4930	0.0000

Table B.5: Quantile Regression (Whole Sample; Type-II Consumption; $x_t = \log(R^e_{t+1})$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0034	0.0102	0.4000	-0.0013	0.0108	0.4440	0.0008	0.0113	0.4760
0.10	-0.0034	0.0087	0.3840	-0.0021	0.0088	0.4240	-0.0009	0.0089	0.4680
0.15	-0.0028	0.0080	0.3920	-0.0017	0.0080	0.4240	-0.0007	0.0080	0.4680
0.20	-0.0019	0.0075	0.4080	-0.0010	0.0075	0.4680	-0.0001	0.0075	0.4840
0.25	-0.0010	0.0073	0.4480	-0.0002	0.0072	0.4920	0.0006	0.0072	0.4680
0.30	-0.0001	0.0071	0.4720	0.0007	0.0071	0.4840	0.0015	0.0070	0.4200
0.35	0.0009	0.0071	0.4720	0.0017	0.0070	0.4200	0.0024	0.0070	0.3680
0.40	0.0019	0.0070	0.4000	0.0027	0.0070	0.3400	0.0034	0.0069	0.3160
0.45	0.0030	0.0070	0.3360	0.0038	0.0070	0.2960	0.0044	0.0069	0.2640
0.50	0.0042	0.0071	0.2640	0.0050	0.0070	0.2360	0.0056	0.0070	0.2040
0.55	0.0054	0.0071	0.2200	0.0062	0.0071	0.1760	0.0069	0.0070	0.1600
0.60	0.0068	0.0072	0.1720	0.0076	0.0072	0.1400	0.0083	0.0071	0.1080
0.65	0.0084	0.0074	0.1240	0.0092	0.0073	0.1040	0.0099	0.0073	0.0720
0.70	0.0101	0.0076	0.0840	0.0111	0.0076	0.0600	0.0119	0.0075	0.0520
0.75	0.0122	0.0079	0.0520	0.0133	0.0079	0.0440	0.0142	0.0078	0.0400
0.80	0.0148	0.0083	0.0440	0.0161	0.0083	0.0280	0.0171	0.0083	0.0200
0.85	0.0181	0.0090	0.0160	0.0199	0.0090	0.0080	0.0212	0.0090	0.0080
0.90	0.0230	0.0101	0.0080	0.0255	0.0103	0.0080	0.0276	0.0104	0.0040
0.95	0.0313	0.0127	0.0120	0.0365	0.0137	0.0040	0.0413	0.0145	0.0040

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	0.0026	0.0117	0.4200	0.0041	0.0119	0.3840	0.0053	0.0121	0.3600
0.10	0.0002	0.0090	0.5000	0.0011	0.0090	0.4480	0.0018	0.0090	0.4240
0.15	0.0001	0.0080	0.4920	0.0008	0.0080	0.4440	0.0014	0.0080	0.4280
0.20	0.0006	0.0075	0.4680	0.0012	0.0075	0.4320	0.0017	0.0075	0.4080
0.25	0.0013	0.0072	0.4360	0.0018	0.0072	0.4040	0.0022	0.0072	0.3880
0.30	0.0021	0.0070	0.3840	0.0026	0.0070	0.3720	0.0030	0.0070	0.3560
0.35	0.0030	0.0069	0.3360	0.0034	0.0069	0.3240	0.0038	0.0069	0.3120
0.40	0.0039	0.0069	0.2880	0.0044	0.0069	0.2720	0.0047	0.0069	0.2480
0.45	0.0050	0.0069	0.2240	0.0054	0.0069	0.2040	0.0058	0.0069	0.1920
0.50	0.0061	0.0069	0.1800	0.0066	0.0069	0.1640	0.0069	0.0069	0.1480
0.55	0.0074	0.0070	0.1400	0.0078	0.0070	0.1280	0.0082	0.0070	0.1120
0.60	0.0089	0.0071	0.1040	0.0093	0.0071	0.0880	0.0096	0.0071	0.0800
0.65	0.0105	0.0073	0.0560	0.0109	0.0072	0.0560	0.0113	0.0072	0.0440
0.70	0.0125	0.0075	0.0400	0.0129	0.0075	0.0400	0.0133	0.0074	0.0400
0.75	0.0148	0.0078	0.0360	0.0153	0.0078	0.0280	0.0157	0.0077	0.0200
0.80	0.0179	0.0083	0.0200	0.0185	0.0082	0.0160	0.0189	0.0082	0.0160
0.85	0.0222	0.0090	0.0080	0.0229	0.0090	0.0080	0.0234	0.0089	0.0040
0.90	0.0291	0.0104	0.0040	0.0303	0.0104	0.0000	0.0310	0.0104	0.0000
0.95	0.0454	0.0149	0.0040	0.0484	0.0150	0.0040	0.0504	0.0150	0.0040

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	0.0061	0.0122	0.3400	0.0066	0.0122	0.3240	0.0069	0.0122	0.3200
0.10	0.0023	0.0090	0.4080	0.0026	0.0090	0.3920	0.0028	0.0090	0.3800
0.15	0.0018	0.0080	0.4120	0.0021	0.0080	0.3960	0.0022	0.0080	0.3920
0.20	0.0020	0.0074	0.3920	0.0023	0.0074	0.3880	0.0024	0.0074	0.3840
0.25	0.0026	0.0072	0.3800	0.0028	0.0071	0.3720	0.0029	0.0071	0.3560
0.30	0.0033	0.0070	0.3280	0.0035	0.0070	0.3280	0.0036	0.0070	0.3200
0.35	0.0041	0.0069	0.2920	0.0043	0.0069	0.2840	0.0044	0.0069	0.2760
0.40	0.0050	0.0068	0.2400	0.0052	0.0068	0.2320	0.0053	0.0068	0.2240
0.45	0.0060	0.0068	0.1840	0.0062	0.0068	0.1840	0.0063	0.0068	0.1720
0.50	0.0072	0.0069	0.1400	0.0073	0.0069	0.1320	0.0074	0.0069	0.1320
0.55	0.0084	0.0070	0.1000	0.0086	0.0069	0.0960	0.0087	0.0069	0.0880
0.60	0.0099	0.0071	0.0760	0.0100	0.0071	0.0600	0.0101	0.0071	0.0560
0.65	0.0115	0.0072	0.0440	0.0117	0.0072	0.0440	0.0118	0.0072	0.0440
0.70	0.0135	0.0074	0.0360	0.0137	0.0074	0.0360	0.0138	0.0074	0.0360
0.75	0.0160	0.0077	0.0200	0.0161	0.0077	0.0200	0.0162	0.0077	0.0200
0.80	0.0192	0.0082	0.0080	0.0194	0.0082	0.0080	0.0195	0.0082	0.0080
0.85	0.0238	0.0089	0.0040	0.0240	0.0089	0.0040	0.0241	0.0089	0.0040
0.90	0.0315	0.0103	0.0000	0.0319	0.0103	0.0000	0.0320	0.0103	0.0000
0.95	0.0516	0.0148	0.0040	0.0522	0.0147	0.0000	0.0525	0.0146	0.0000

Table B.6: Quantile Regression (Whole Sample; Type-II Consumption; $x_t = \log(R_t^f)$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.2683	0.1005	0.0040	-0.3005	0.1065	0.0000	-0.3125	0.1120	0.0000
0.10	-0.0239	0.0850	0.3920	-0.0236	0.0865	0.3960	-0.0132	0.0876	0.4360
0.15	0.1421	0.0772	0.0280	0.1522	0.0774	0.0200	0.1676	0.0775	0.0080
0.20	0.2724	0.0725	0.0000	0.2868	0.0722	0.0000	0.3036	0.0720	0.0000
0.25	0.3832	0.0695	0.0000	0.3998	0.0690	0.0000	0.4172	0.0686	0.0000
0.30	0.4826	0.0675	0.0000	0.5006	0.0668	0.0000	0.5180	0.0664	0.0000
0.35	0.5751	0.0661	0.0000	0.5943	0.0655	0.0000	0.6118	0.0650	0.0000
0.40	0.6640	0.0653	0.0000	0.6840	0.0646	0.0000	0.7017	0.0641	0.0000
0.45	0.7514	0.0649	0.0000	0.7725	0.0642	0.0000	0.7904	0.0636	0.0000
0.50	0.8395	0.0648	0.0000	0.8617	0.0641	0.0000	0.8800	0.0636	0.0000
0.55	0.9303	0.0651	0.0000	0.9540	0.0644	0.0000	0.9729	0.0639	0.0000
0.60	1.0259	0.0658	0.0000	1.0516	0.0650	0.0000	1.0714	0.0646	0.0000
0.65	1.1292	0.0669	0.0000	1.1575	0.0662	0.0000	1.1789	0.0657	0.0000
0.70	1.2437	0.0684	0.0000	1.2759	0.0678	0.0000	1.2995	0.0673	0.0000
0.75	1.3751	0.0707	0.0000	1.4132	0.0702	0.0000	1.4405	0.0698	0.0000
0.80	1.5327	0.0740	0.0000	1.5805	0.0738	0.0000	1.6138	0.0736	0.0000
0.85	1.7336	0.0792	0.0000	1.7987	0.0794	0.0000	1.8441	0.0795	0.0000
0.90	2.0159	0.0879	0.0000	2.1189	0.0895	0.0000	2.1924	0.0906	0.0000
0.95	2.4939	0.1069	0.0000	2.7190	0.1144	0.0000	2.9018	0.1209	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.3069	0.1164	0.0080	-0.2892	0.1195	0.0160	-0.2660	0.1213	0.0200
0.10	0.0034	0.0883	0.4880	0.0225	0.0887	0.4080	0.0411	0.0888	0.3440
0.15	0.1851	0.0775	0.0040	0.2025	0.0774	0.0040	0.2182	0.0773	0.0000
0.20	0.3207	0.0717	0.0000	0.3367	0.0716	0.0000	0.3505	0.0714	0.0000
0.25	0.4337	0.0682	0.0000	0.4484	0.0681	0.0000	0.4609	0.0679	0.0000
0.30	0.5340	0.0660	0.0000	0.5479	0.0658	0.0000	0.5594	0.0656	0.0000
0.35	0.6273	0.0646	0.0000	0.6403	0.0644	0.0000	0.6511	0.0642	0.0000
0.40	0.7168	0.0637	0.0000	0.7293	0.0635	0.0000	0.7394	0.0633	0.0000
0.45	0.8052	0.0633	0.0000	0.8173	0.0631	0.0000	0.8269	0.0629	0.0000
0.50	0.8948	0.0632	0.0000	0.9066	0.0630	0.0000	0.9157	0.0629	0.0000
0.55	0.9878	0.0635	0.0000	0.9993	0.0633	0.0000	1.0081	0.0632	0.0000
0.60	1.0867	0.0642	0.0000	1.0981	0.0640	0.0000	1.1066	0.0638	0.0000
0.65	1.1947	0.0654	0.0000	1.2062	0.0651	0.0000	1.2144	0.0650	0.0000
0.70	1.3164	0.0670	0.0000	1.3283	0.0668	0.0000	1.3365	0.0667	0.0000
0.75	1.4593	0.0695	0.0000	1.4720	0.0693	0.0000	1.4803	0.0692	0.0000
0.80	1.6362	0.0734	0.0000	1.6505	0.0732	0.0000	1.6593	0.0731	0.0000
0.85	1.8736	0.0796	0.0000	1.8916	0.0795	0.0000	1.9017	0.0794	0.0000
0.90	2.2403	0.0913	0.0000	2.2683	0.0916	0.0000	2.2828	0.0917	0.0000
0.95	3.0316	0.1255	0.0000	3.1096	0.1282	0.0000	3.1474	0.1295	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\vert\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.2428	0.1221	0.0320	-0.2239	0.1224	0.0400	-0.2119	0.1225	0.0520
0.10	0.0570	0.0887	0.2680	0.0690	0.0887	0.2200	0.0764	0.0886	0.1920
0.15	0.2311	0.0772	0.0000	0.2406	0.0772	0.0000	0.2464	0.0771	0.0000
0.20	0.3616	0.0713	0.0000	0.3697	0.0712	0.0000	0.3745	0.0712	0.0000
0.25	0.4708	0.0678	0.0000	0.4779	0.0677	0.0000	0.4822	0.0677	0.0000
0.30	0.5683	0.0655	0.0000	0.5748	0.0655	0.0000	0.5786	0.0654	0.0000
0.35	0.6594	0.0641	0.0000	0.6652	0.0640	0.0000	0.6687	0.0640	0.0000
0.40	0.7471	0.0632	0.0000	0.7525	0.0632	0.0000	0.7557	0.0631	0.0000
0.45	0.8341	0.0628	0.0000	0.8391	0.0628	0.0000	0.8421	0.0627	0.0000
0.50	0.9225	0.0627	0.0000	0.9272	0.0627	0.0000	0.9299	0.0627	0.0000
0.55	1.0145	0.0631	0.0000	1.0189	0.0630	0.0000	1.0214	0.0630	0.0000
0.60	1.1126	0.0637	0.0000	1.1167	0.0637	0.0000	1.1191	0.0637	0.0000
0.65	1.2202	0.0649	0.0000	1.2240	0.0648	0.0000	1.2262	0.0648	0.0000
0.70	1.3420	0.0666	0.0000	1.3455	0.0665	0.0000	1.3474	0.0665	0.0000
0.75	1.4855	0.0691	0.0000	1.4888	0.0690	0.0000	1.4904	0.0690	0.0000
0.80	1.6645	0.0729	0.0000	1.6673	0.0728	0.0000	1.6686	0.0728	0.0000
0.85	1.9070	0.0793	0.0000	1.9094	0.0792	0.0000	1.9104	0.0792	0.0000
0.90	2.2890	0.0916	0.0000	2.2910	0.0915	0.0000	2.2912	0.0915	0.0000
0.95	3.1601	0.1296	0.0000	3.1604	0.1295	0.0000	3.1572	0.1294	0.0000

Table B.7: Quantile Regression (Young Households; Type-II Consumption; $x_t = \log(R_{t+1})$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0574	0.0264	0.0040	-0.0577	0.0283	0.0120	-0.0576	0.0299	0.0120
0.10	-0.0515	0.0223	0.0040	-0.0515	0.0229	0.0040	-0.0511	0.0233	0.0040
0.15	-0.0463	0.0202	0.0040	-0.0459	0.0203	0.0040	-0.0454	0.0204	0.0040
0.20	-0.0418	0.0188	0.0040	-0.0412	0.0188	0.0040	-0.0406	0.0188	0.0040
0.25	-0.0377	0.0179	0.0040	-0.0370	0.0178	0.0040	-0.0363	0.0177	0.0040
0.30	-0.0340	0.0172	0.0080	-0.0332	0.0170	0.0080	-0.0324	0.0169	0.0080
0.35	-0.0304	0.0166	0.0160	-0.0295	0.0165	0.0200	-0.0287	0.0163	0.0320
0.40	-0.0269	0.0163	0.0640	-0.0260	0.0160	0.0640	-0.0251	0.0159	0.0640
0.45	-0.0234	0.0160	0.0800	-0.0224	0.0157	0.0920	-0.0216	0.0156	0.0960
0.50	-0.0198	0.0158	0.1160	-0.0188	0.0155	0.1240	-0.0179	0.0154	0.1400
0.55	-0.0162	0.0157	0.1560	-0.0150	0.0154	0.1640	-0.0141	0.0153	0.1840
0.60	-0.0123	0.0157	0.2120	-0.0110	0.0154	0.2440	-0.0100	0.0152	0.2640
0.65	-0.0081	0.0158	0.3120	-0.0067	0.0155	0.3280	-0.0056	0.0153	0.3520
0.70	-0.0035	0.0160	0.4000	-0.0018	0.0157	0.4600	-0.0006	0.0156	0.4960
0.75	0.0019	0.0164	0.4600	0.0039	0.0162	0.4160	0.0053	0.0160	0.3760
0.80	0.0082	0.0171	0.3080	0.0107	0.0169	0.2720	0.0126	0.0168	0.2400
0.85	0.0163	0.0183	0.1920	0.0197	0.0183	0.1600	0.0222	0.0183	0.1200
0.90	0.0276	0.0207	0.1040	0.0327	0.0212	0.0640	0.0366	0.0215	0.0360
0.95	0.0507	0.0330	0.0560	0.0589	0.0338	0.0440	0.0653	0.0354	0.0360

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0573	0.0311	0.0200	-0.0569	0.0318	0.0280	-0.0565	0.0322	0.0280
0.10	-0.0506	0.0235	0.0040	-0.0499	0.0236	0.0080	-0.0493	0.0237	0.0080
0.15	-0.0447	0.0205	0.0040	-0.0440	0.0205	0.0040	-0.0434	0.0205	0.0040
0.20	-0.0399	0.0188	0.0040	-0.0392	0.0187	0.0040	-0.0386	0.0187	0.0040
0.25	-0.0356	0.0176	0.0040	-0.0350	0.0176	0.0040	-0.0344	0.0175	0.0080
0.30	-0.0317	0.0168	0.0080	-0.0311	0.0168	0.0160	-0.0306	0.0167	0.0160
0.35	-0.0280	0.0162	0.0320	-0.0274	0.0162	0.0320	-0.0269	0.0161	0.0360
0.40	-0.0244	0.0158	0.0640	-0.0238	0.0157	0.0800	-0.0234	0.0157	0.0880
0.45	-0.0208	0.0155	0.1040	-0.0203	0.0154	0.1080	-0.0198	0.0154	0.1080
0.50	-0.0172	0.0153	0.1400	-0.0166	0.0152	0.1480	-0.0161	0.0152	0.1600
0.55	-0.0133	0.0151	0.1920	-0.0128	0.0151	0.2000	-0.0123	0.0150	0.2000
0.60	-0.0092	0.0151	0.2720	-0.0086	0.0151	0.2880	-0.0082	0.0150	0.3000
0.65	-0.0047	0.0152	0.3680	-0.0041	0.0151	0.3960	-0.0037	0.0151	0.4080
0.70	0.0004	0.0155	0.4800	0.0010	0.0154	0.4720	0.0015	0.0153	0.4640
0.75	0.0064	0.0159	0.3600	0.0071	0.0158	0.3400	0.0076	0.0158	0.3280
0.80	0.0139	0.0167	0.2120	0.0147	0.0167	0.1960	0.0153	0.0167	0.1880
0.85	0.0239	0.0183	0.1080	0.0251	0.0183	0.0920	0.0258	0.0183	0.0880
0.90	0.0394	0.0218	0.0320	0.0412	0.0219	0.0240	0.0424	0.0219	0.0240
0.95	0.0708	0.0356	0.0240	0.0753	0.0357	0.0240	0.0785	0.0354	0.0160

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-0.0561	0.0323	0.0320	-0.0558	0.0323	0.0320	-0.0556	0.0323	0.0320
0.10	-0.0488	0.0237	0.0080	-0.0483	0.0236	0.0080	-0.0481	0.0236	0.0080
0.15	-0.0429	0.0204	0.0040	-0.0425	0.0204	0.0080	-0.0423	0.0204	0.0080
0.20	-0.0381	0.0187	0.0040	-0.0378	0.0186	0.0040	-0.0376	0.0186	0.0040
0.25	-0.0340	0.0175	0.0080	-0.0337	0.0175	0.0080	-0.0335	0.0175	0.0080
0.30	-0.0302	0.0167	0.0240	-0.0299	0.0167	0.0240	-0.0297	0.0167	0.0240
0.35	-0.0265	0.0161	0.0360	-0.0263	0.0161	0.0360	-0.0261	0.0161	0.0360
0.40	-0.0230	0.0157	0.0960	-0.0227	0.0156	0.0960	-0.0226	0.0156	0.0920
0.45	-0.0195	0.0153	0.1080	-0.0192	0.0153	0.1160	-0.0191	0.0153	0.1160
0.50	-0.0158	0.0151	0.1600	-0.0156	0.0151	0.1560	-0.0154	0.0151	0.1560
0.55	-0.0120	0.0150	0.2040	-0.0118	0.0150	0.2200	-0.0116	0.0150	0.2280
0.60	-0.0079	0.0150	0.3080	-0.0077	0.0150	0.3120	-0.0075	0.0149	0.3160
0.65	-0.0033	0.0151	0.4200	-0.0031	0.0150	0.4280	-0.0030	0.0150	0.4320
0.70	0.0018	0.0153	0.4480	0.0020	0.0153	0.4480	0.0022	0.0153	0.4440
0.75	0.0080	0.0158	0.3200	0.0082	0.0158	0.3160	0.0083	0.0157	0.3080
0.80	0.0157	0.0166	0.1800	0.0159	0.0166	0.1760	0.0161	0.0166	0.1760
0.85	0.0263	0.0182	0.0800	0.0266	0.0182	0.0800	0.0267	0.0182	0.0800
0.90	0.0431	0.0219	0.0240	0.0435	0.0218	0.0240	0.0436	0.0218	0.0240
0.95	0.0805	0.0350	0.0080	0.0817	0.0347	0.0040	0.0823	0.0344	0.0040

Table B.8: Quantile Regression (Young Households; Type-II Consumption; $x_t = \log(R_t^f)$)

	$\sigma_\eta = 0.9\sigma_\varepsilon$			$\sigma_\eta = 0.8\sigma_\varepsilon$			$\sigma_\eta = 0.7\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-1.1832	0.2349	0.0000	-1.2930	0.2520	0.0000	-1.3726	0.2698	0.0000
0.10	-0.8496	0.2023	0.0000	-0.8921	0.2069	0.0000	-0.9131	0.2110	0.0000
0.15	-0.6233	0.1857	0.0000	-0.6427	0.1866	0.0000	-0.6483	0.1874	0.0000
0.20	-0.4475	0.1757	0.0080	-0.4561	0.1751	0.0040	-0.4555	0.1746	0.0040
0.25	-0.3000	0.1695	0.0440	-0.3027	0.1680	0.0400	-0.2993	0.1669	0.0400
0.30	-0.1700	0.1654	0.1600	-0.1690	0.1635	0.1600	-0.1640	0.1621	0.1600
0.35	-0.0508	0.1628	0.3880	-0.0473	0.1606	0.3960	-0.0415	0.1591	0.4200
0.40	0.0616	0.1614	0.3640	0.0670	0.1591	0.3480	0.0733	0.1574	0.3480
0.45	0.1704	0.1608	0.1600	0.1774	0.1584	0.1480	0.1841	0.1567	0.1320
0.50	0.2780	0.1611	0.0360	0.2866	0.1586	0.0320	0.2938	0.1568	0.0200
0.55	0.3870	0.1621	0.0000	0.3973	0.1596	0.0000	0.4050	0.1577	0.0000
0.60	0.4997	0.1639	0.0000	0.5121	0.1613	0.0000	0.5204	0.1594	0.0000
0.65	0.6191	0.1666	0.0000	0.6342	0.1641	0.0000	0.6436	0.1622	0.0000
0.70	0.7490	0.1704	0.0000	0.7678	0.1680	0.0000	0.7791	0.1662	0.0000
0.75	0.8949	0.1758	0.0000	0.9195	0.1736	0.0000	0.9338	0.1720	0.0000
0.80	1.0659	0.1835	0.0000	1.1000	0.1819	0.0000	1.1196	0.1806	0.0000
0.85	1.2784	0.1952	0.0000	1.3297	0.1949	0.0000	1.3603	0.1945	0.0000
0.90	1.5679	0.2156	0.0000	1.6572	0.2187	0.0000	1.7147	0.2208	0.0000
0.95	2.0492	0.2867	0.0000	2.2557	0.2897	0.0000	2.4200	0.2990	0.0000

	$\sigma_\eta = 0.6\sigma_\varepsilon$			$\sigma_\eta = 0.5\sigma_\varepsilon$			$\sigma_\eta = 0.4\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-1.4194	0.2861	0.0000	-1.4367	0.2991	0.0000	-1.4326	0.3077	0.0000
0.10	-0.9175	0.2141	0.0000	-0.9109	0.2162	0.0000	-0.8988	0.2175	0.0000
0.15	-0.6443	0.1880	0.0000	-0.6353	0.1883	0.0000	-0.6243	0.1884	0.0000
0.20	-0.4493	0.1744	0.0040	-0.4403	0.1741	0.0040	-0.4308	0.1739	0.0040
0.25	-0.2923	0.1662	0.0400	-0.2840	0.1657	0.0400	-0.2758	0.1654	0.0400
0.30	-0.1571	0.1612	0.1720	-0.1496	0.1606	0.1760	-0.1425	0.1601	0.1840
0.35	-0.0348	0.1581	0.4440	-0.0281	0.1573	0.4560	-0.0220	0.1568	0.4600
0.40	0.0797	0.1562	0.3280	0.0857	0.1555	0.3120	0.0909	0.1549	0.3080
0.45	0.1902	0.1555	0.1120	0.1955	0.1546	0.0960	0.1999	0.1540	0.0920
0.50	0.2996	0.1555	0.0160	0.3042	0.1546	0.0120	0.3078	0.1540	0.0120
0.55	0.4105	0.1564	0.0000	0.4145	0.1554	0.0000	0.4174	0.1548	0.0000
0.60	0.5259	0.1581	0.0000	0.5293	0.1571	0.0000	0.5314	0.1564	0.0000
0.65	0.6491	0.1608	0.0000	0.6521	0.1598	0.0000	0.6535	0.1591	0.0000
0.70	0.7850	0.1648	0.0000	0.7877	0.1638	0.0000	0.7883	0.1630	0.0000
0.75	0.9408	0.1707	0.0000	0.9433	0.1696	0.0000	0.9433	0.1688	0.0000
0.80	1.1291	0.1795	0.0000	1.1320	0.1785	0.0000	1.1313	0.1778	0.0000
0.85	1.3754	0.1939	0.0000	1.3801	0.1933	0.0000	1.3790	0.1927	0.0000
0.90	1.7456	0.2219	0.0000	1.7572	0.2224	0.0000	1.7572	0.2222	0.0000
0.95	2.5254	0.3054	0.0000	2.5781	0.3105	0.0000	2.5917	0.3137	0.0000

	$\sigma_\eta = 0.3\sigma_\varepsilon$			$\sigma_\eta = 0.2\sigma_\varepsilon$			$\sigma_\eta = 0.1\sigma_\varepsilon$		
	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$	$\delta_1(\tau\|\sigma_\eta)$		$sign(\delta_1)$
$\tau =$	coef.	s.e.	p-value	coef.	s.e.	p-value	coef.	s.e.	p-value
0.05	-1.4175	0.3125	0.0000	-1.4003	0.3148	0.0000	-1.3877	0.3156	0.0000
0.10	-0.8858	0.2182	0.0000	-0.8745	0.2184	0.0000	-0.8672	0.2185	0.0000
0.15	-0.6138	0.1885	0.0000	-0.6054	0.1884	0.0000	-0.6000	0.1884	0.0000
0.20	-0.4222	0.1737	0.0040	-0.4155	0.1735	0.0040	-0.4113	0.1735	0.0040
0.25	-0.2687	0.1651	0.0520	-0.2632	0.1649	0.0560	-0.2598	0.1648	0.0600
0.30	-0.1365	0.1598	0.1920	-0.1319	0.1596	0.2040	-0.1292	0.1594	0.2080
0.35	-0.0169	0.1565	0.4680	-0.0132	0.1563	0.4760	-0.0109	0.1561	0.4760
0.40	0.0951	0.1545	0.3000	0.0982	0.1543	0.2920	0.1000	0.1541	0.2840
0.45	0.2033	0.1536	0.0800	0.2058	0.1534	0.0800	0.2072	0.1532	0.0760
0.50	0.3105	0.1536	0.0080	0.3124	0.1533	0.0040	0.3135	0.1531	0.0040
0.55	0.4194	0.1543	0.0000	0.4207	0.1541	0.0000	0.4214	0.1539	0.0000
0.60	0.5328	0.1560	0.0000	0.5335	0.1556	0.0000	0.5339	0.1555	0.0000
0.65	0.6541	0.1585	0.0000	0.6542	0.1582	0.0000	0.6541	0.1580	0.0000
0.70	0.7881	0.1625	0.0000	0.7875	0.1621	0.0000	0.7870	0.1619	0.0000
0.75	0.9421	0.1683	0.0000	0.9407	0.1679	0.0000	0.9397	0.1676	0.0000
0.80	1.1291	0.1772	0.0000	1.1266	0.1768	0.0000	1.1249	0.1766	0.0000
0.85	1.3754	0.1921	0.0000	1.3715	0.1917	0.0000	1.3687	0.1914	0.0000
0.90	1.7518	0.2219	0.0000	1.7453	0.2215	0.0000	1.7404	0.2214	0.0000
0.95	2.5836	0.3150	0.0000	2.5683	0.3154	0.0000	2.5557	0.3153	0.0000

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